A problem of the Γ-conditionalization account

1 Γ-conditionalization account of non-logical analyticity

Let me call a statement analytic if it is either self-validating or self-contradictory. And, if the analyticity of an analytic statement is obtained already at the level of logic, let me say that it is logically analytic. Accordingly, let me call a statement non-logically analytic if it is analytic but its analyticity does not obtain at the logical level. For instance, "For all $x$, either $x = x$ or $x \neq x$" --- as well as its negation, for that matter --- is presumably logically analytic. In contrast, "For all $x$, $x = x$" and its negation are presumably non-logically analytic (for those of us who know what the symbol '=' means).

I have offered an account of non-logical analyticity in this "blog," calling it the "Γ-conditionalization" account.1 The account goes like this (with minor revisions for the sake of later discussion):

(Γ1) Γ-conditionalization account of non-logical analyticity

Let $p$ be a statement of a language $L$ which is not logically analytic. Still, $p$ is analytic in $L$ if there is a concept-defining structurality $Γ$ built in $L$ such that

If $Γ$, then $p$ is logically analytic.

This account (Γ1) assumes that a language $L$ can contain a multitude of concept-defining structuralities. I made (Γ1) this way primarily because a natural language obviously seems this way. For instance, I think both of "If something is right, then it's not wrong" and "If something is on your right, then it's not on your left" can be regarded as non-logically analytic in English, because their respective truths are encoded in the semantics of English language, in particular, in the semantics of the "right/wrong" distinction and the "right/left" distinction, respectively. But, these semantic distinctions apparently belong to two different structuralities, say $Γ$ and $Γ'$, both constituting part of the whole semantics of English. So, the former is analytic with respect to $Γ$ (but not with respect to $Γ'$), while the latter is analytic with respect to $Γ'$ (but not $Γ$).

To accommodate this multifarious structurality, let me adopt further terminological convention. Let me say that an $L$-statement $p$ is $Γ$-analytic in order to specify that $p$ is non-logically analytic specifically with respect to some particular $Γ$ (built in $L$, of course). I shall also say that $p$ is $Γ$-analytically true or false to imply that $p$ is $Γ$-analytically self-validating or self-contradictory, respectively.

1 More precisely, the account has been offered through two posts, first in part (A) of "Even Quine as committed to the 'two assumptions'," in which it was first offered in its all essence, though without a clear formulation, and then in "Synonym-replacement account reconsidered," in which the account was clearly formulated. I have also called it the "axiomatic account," in a revised version of the part (A) of the "Even Quine ...".
2 Γ-equivalence account of synonymy and its problem

Now, it seems that this account of analyticity, (Γ1), naturally suggests an account of statement-synonymy, in the following way.

(Γ2) Γ-equivalence account of statement-synonymy

Let each of p and q denote a statement of a language L. Then, p and q are synonymous in L just in case there is a concept-defining structurality Γ built in L such that

\[ p \text{ if and only if } q \]

is Γ-analytically true.

One more terminological note: Let me say that such synonymous statements p and q are Γ-synonymous (or Γ-equivalent) to specify the particular structurality Γ which is responsible for their synonymy.

The other day, it occurred to me that this account of statement-synonymy, natural as it may be, would give rise to a philosophical problem. It seemed to me that, if two L-statements p and q were Γ-synonymous, that would have to mean that their "meanings" were indiscernible from the perspective of Γ. But, it's not uncommon that we do discern some differences in the "meanings" of two distinct but Γ-synonymous statements p and q even when we apparently take them from the perspective of Γ.

To illustrate this problem in a clearest form, let me draw a semi-artificial example. Let L now denote an artificially instituted "dialect" of a natural language (such as English). I mean, let L denote a semi-natural and semi-artificial "language," which is temporarily instituted by someone's stipulating a certain explicitly stated axiomatic definition Γ of an abstract mathematical concept --- and, importantly, by someone else's accepting it --- in effect as the rule-of-inference and precondition-of-interpretation of that "language." For example, we start using such an L when we encounter a definition Γ of the concept of group in a textbook. At that moment, there starts a temporary and miniature-scale linguistic convention L, taking effect between the textbook author (who stipulates Γ) and we (the readers who accept Γ, including the author as she reads

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2 By assuming that the "one" who stipulates and the "one" who accepts Γ are distinct (by using the phrase "someone else"), I assume a sort of personhood constructionism here, which is a part of (or, which includes as its part) my diachronic and structural-pragmatist view of language. (I'm talking about the view of language I presented in a PPT presentation.) According to this view of language, language is a diachronic (historical) and social phenomenon whose existence or reality consists in our uses of language (which too are historical and social phenomena). Seen in this way, our uses of language are not just "consumptive" moments of using something which is already there for us to use, but are also "productive/reproductive" moments of letting it be there: moments of inheriting (from preceding generations of uses of the language) and passing on (to succeeding generations thereof) much of its components while possibly revising some others. My personhood constructionism further holds that moments of our language-use are similarly both "consumptive" and "productive/reproductive" with respect to our own beings, too. They are not only moments of manifestation of our otherwise latently chronologically continuing beings, but are also moments of letting us (the chronologically self-identical beings) be here. The chronologically self-identical "person" --- even the one assumed in Cartesian "I think therefore I am" ---- is as much a product of the historical-social phenomenon of language, and, in that sense, as much a semantic concept, as any semantic concepts encoded in a language. According to this view, the stipulator and the acceptor of Γ may or may not be the same if we identify them at the semantic-conceptual level. But, they are by definition different if we identify them at the pragmatic level. (Please see my "Deduction is dialogical" for a related discussion.)
As long as we use such an L (i.e., until otherwise instructed by the textbook), we are linguistically bound by Γ (as the rule-of-inference/precondition-of-interpretation of L) as to how (and how not) to make use of Γ-theoretical concepts (e.g., those of the group operation, an identity element, an inverse of an element, and even equality --- in the case of group theory) in actually making and taking inferential and (often hypothetical or fictional) referential claims (or thoughts). In such a situation, according to the account (Γ2), all of the Γ-analytically true L-statements (that is, Γ-theorems) are Γ-synonymous to one another. So too are all the Γ-analytically false L-statements (that is, negations of Γ-theorems and their Γ-synonyms --- which I shall call Γ-contradictions, hereafter). But, we distinguish the "meanings" of some Γ-theorems, as well as those of some Γ-contradictions. Consider group theoretic theorems stating the existence and the uniqueness of identity, for example. They are both group-theorems, but their "meanings," or conceptual contents, clearly differ. Similarly for the statements of the non-existence and the non-uniqueness of identity. Although they are group-theoretically synonymous (in being group-theoretical contradictions) and we know that, they still carry different and relevant "meanings" for us, even when we make/take them as statements of group theory.

An analogous problem arises from an equally natural formulation of the "Γ-equivalence" account of term-synonymy, (Γ3), below.

(Γ3)  Γ-equivalence account of term-synonymy

Let each of t and s denote a term, with the same free variables, $x_1, x_2, ..., x_n$, of a language L. Then, t and s are synonymous in L just in case there is a concept-defining structurality Γ built in L such that

$\text{For all } x_1, x_2, ..., x_n, t = s$

is Γ-analytically true.

(Note: It may or may not be that $n = 0$.)

The problem must be obvious to anyone who (i) accepts that two predicative phrases, "is an identity element" (i.e., "is such an x that $xa = a = ax$ for all a") and "is self-inverse" (i.e., "is such an x that $xx = e$ is for some identity element e") are group-theoretically synonymous (in the sense defined above), and yet (ii) sees that these two phrases convey two different "meanings" even in context of group theory.

It seems that a proponent of the Γ-equivalence account of synonymy has to explain why we can distinguish the "meanings" of statements or terms of L which we know to be Γ-synonymous, even when we operate in L qua Γ-defined language.

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3 This pragmatically based view of mathematical discourse is originally outlined by Kuklitka (2014).
3 Monocularism of conceptual-term application

Facing this problem, I reflected on my own thoughts. I now think I have assumed three increasingly presupposing theses, behind this problem:

I. **Structuralism (of conceptual-term application):** The thesis that whenever we *apply* (i.e., make a use of or take such a use of) a conceptual term of a language L in an actual "discourse" (including even a "monological" one made in mind), we identify the contextually relevant conceptual content (i.e., the "meaning") of the term by locating it in a certain concept-defining structurality Γ built in L.

Notice that to locate the "meaning" of a term one must identify or choose a certain Γ from among many Γ's available in L.

II. **Structure/perspective mutuality (in conceptual-term application):** The thesis that to identify a concept-defining structurality Γ in L, at a moment of actual application of a conceptual term of L, is to take the perspective of Γ at that pragmatic moment.

III. **Monocularism (of conceptual-term application):** The thesis that we cannot take more than one such perspective simultaneously at the same moment of conceptual-term application.

These Theses are increasingly presupposing in that Thesis III presupposes Thesis II, which in turn presupposes Thesis I. Thesis I is almost a paraphrase of the theory-laden view of language which motivates the Γ-conditionalization account of analyticity. Thesis II is a core of a kind of personhood constructionism. (See the footnote 2 for more on this constructionism.) Both Theses are central to my "re-thinking of mind" project (the theme of this website). I see no strong reason to revise them, at least as of now. In fact, even Thesis III seems to be correct, if it’s examined in certain contexts. Recall "If it’s right, then it’s not wrong" and "If it’s on your right, then it’s not on your left," for example. Between two contexts of making/taking of the former and the latter statements, we use the conceptual term "right" differently because we invoke, as it were, the two different concept-defining structuralities (Γ and Γ') in applying the morphologically same term. This is merely to say that we take different perspectives in applying the term in the two contexts.

Now, if I’m right, we cannot take an occurrence of the term "right" *both* in the evaluative sense belonging to Γ and in the spatial-orientational sense belonging to Γ' *simultaneously*, at a single moment of its application. Presumably, this is because we cannot take two distinct perspectives simultaneously in actually applying a conceptual term. Our conceptual "mind’s eye" seems to be "monocular," not "compound-eye."

However, when it comes to the examples of the two theorems of group theory ("There is an identity" and "An identity is unique") or two synonymous group theoretic terms ("is an identity" and "is a self-inverse"), our ability to perceive their difference in "meaning" calls for a reassessment of the Thesis III. It seems to hold in some cases, but not all. When does it hold, and when not?
Vertically "compound-eye" view of conceptual-term application

Here is my tentative answer to that question.

It may be that when we detect "semantic differences" among some \( \Gamma \)-synonymous statements or terms of \( L \) (where \( \Gamma \) is built in \( L \)) while knowing that they are \( \Gamma \)-synonymous, that part of us which differentiates them perceives them from the perspective of a certain concept-defining structurality \( \Gamma' \) which is conceptually less committing than \( \Gamma \). Here, by saying that \( \Gamma' \) is conceptually less committing than \( \Gamma \), I mean that \( \Gamma' \) (as a whole) is properly presupposed by \( \Gamma \) (as a whole) in the sense that all theorems of \( \Gamma' \) are theorems of \( \Gamma \) but not vice versa.

Such \( \Gamma' \) may be an empty axiom-set, representing the perspective of mere predicate logic. But, \( \Gamma' \) may be nonempty. For an illustration of this case, consider \( \Gamma \) to be a standard definition of group. The \( \Gamma \)-defined "dialect" \( L \) would then be a first-order language whose "primitive" (non-logical) lexicon consists of just two items, namely, equality = and group operation \(*\). An example of a structurality \( \Gamma' \) which is conceptually less committing than this \( \Gamma \) is a set of axioms 1, 2, 3, and 4 below,

1. For all \( x \) in \( G \), \( x = x \);
2. For all \( x, y \) in \( G \), if \( x = y \), then \( y = x \);
3. For all \( x, y, z \) in \( G \), if \( x = y \) and \( y = z \), then \( x = z \); and
4. For all \( x, y, z \) in \( G \), if \( x = y \), then \( x * z = y * z \) and \( z * x = z * y \).

Let me denote it as \( \Gamma_1 \). Another example would be a subset \( \Gamma_2 = \{1, 2, 3\} \) of \( \Gamma_1 \). \( \Gamma_2 \) just defines (in the sense of stipulation) that its equality = be an equivalence relation on \( G \). (And, by the way, the \( \Gamma_2 \)-defined "dialect" \( L \) contains only one item in its "primitive" lexicon, namely, equality =) \( \Gamma_1 \), which is an extension of \( \Gamma_2 \), is a little more conceptually committing than \( \Gamma_2 \) in that it stipulates that its = be a special kind of equivalence relation such that any two open terms\(^5\) which are \( \Gamma_1 \)-synonymous (in the sense of \( (\Gamma 5) \) above) are substitutable without changing the \( \Gamma_1 \)-theoretical "meaning" (or intension) of the context of substitution.\(^6\)

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\(^4\) Here I'm assuming (without proving yet) that the axiom 4 makes the following two statements theorems-schemata of \( \Gamma_1 = \{1, 2, 3, 4\} \):

a) For all \( x_1, x_2, ..., x_n \) and \( y_1, y_2, ..., y_n \) in \( G \), if \( x_i = y_i \) for each \( i \ (1 \leq i \leq n) \), then, \( t[x] = t[y] \)

   --- where \( t[x] \) is any open term in which no variable occurs free other than \( x_1, x_2, ..., x_n \) \( (n > 0) \), and \( t[y] \) is the result of substituting each \( x_i \) in \( t[x] \) with \( y_i \).

b) For all \( x_1, x_2, ..., x_n \) and \( y_1, y_2, ..., y_n \) in \( G \), if \( x_i = y_i \) for each \( i \ (1 \leq i \leq n) \), then \( A[x] \iff A[y] \)

   --- where \( A[x] \) is any open statement in which no variable occurs free other than \( x_1, x_2, ..., x_n \) \( (n > 0) \), and \( A[y] \) is the result of substituting each \( x_i \) in \( A[x] \) with \( y_i \).

These two are intended to constitute a refined version of what I have called \( \Gamma_1 \)-substitutivity condition. (See here.)

\(^5\) Notice that in the "language" of \( \Gamma_1 \) (or even that of \( \Gamma \)), there are no such things as closed terms.

\(^6\) Here, I'm assuming what may be called the description-substitution interpretation of quantifier. In short, this is a variation of substitution interpretation which assumes that a \( \Gamma \)-defined language \( L \) to which it is applied has no syntactic resource to form a closed variable, and which takes the quantifiers of \( L \) to range over not the class of closed terms (nor any of its subclass) but the class of open terms (with exactly one free variable) of \( L \). In this interpretation, the semantics of "For all \( x, Fx \)" is not to be understood in terms of the extensional (or referential) notion of truth but in terms of the intensional (or inferential) notion of \( \Gamma \)-analyticity, such that: "For all \( x, Fx \)" is \( \Gamma \)-analytically true iff "For all \( y, Py \)" is \( \Gamma \)-analytically true for all open terms \( t \) of \( L \) with exactly one free variable \( y \). (This would surely make a circular definition if it were a definition at all. I think this is just as it should be because the description-substitution interpretation of quantifier is not exactly a way of
Such a context of substitution may be an open term or a statement, either open or closed. I claim that $\Gamma_1$ is conceptually less committing than, that is, properly presupposed by, $\Gamma$ assuming that $\Gamma$ tacitly contains the four axioms of $\Gamma_1$. In fact, judging from mathematicians' actual use of equality = in group theoretical inferential contexts, this assumption seems to fit the reality of mathematical practice. If we further introduce a symbol $\Gamma_3$ to denote the empty axiom-set, then, the four axiom-sets on the table now are, in the order of proper presupposition, such that $\Gamma \rightarrow \Gamma_1 \rightarrow \Gamma_2 \rightarrow \Gamma_3$.

Back to the reassessment of the Thesis III. It may be that, when we work in some "language" (taken in a broad sense of the term) $L$ qua $\Gamma$-defined, our conceptual-term application is not totally "monocular" but at least vertically "compound", i.e., capable of facing a given $\Gamma$-theoretical conceptual situation from a plurality of perspectives, ranging from that of $\Gamma$ itself to that of mere logic, say, $\Gamma_n$, and every possible perspective of $\Gamma_k (0 < k < n)$ in between. If so, then, when we work in such a $\Gamma$-theoretical context, we are capable of applying vertically plural standards of synonymy (i.e., standards of the sameness of intensional "meaning") simultaneously, so that we can sense both the sameness and difference in "meaning" of some $\Gamma$-synonymous (but distinct) statements or terms, simultaneously. And, for that matter, we can also apply vertically plural standards of analyticity (i.e., standards of the intensional "meaningfulness/meaninglessness" distinction) simultaneously, so that we can sense both the "meaningfulness" and "meaninglessness" (i.e., analyticity and non-analyticity) of a statement, simultaneously.

The vertically "compound-eye" view of conceptual-term application can explain our ability to detect both the sameness and the difference in the "meanings" (conceptual contents, or intensions) of the "existence of identity" statement and the "uniqueness of identity" statement in the context of group theory, for example. However, having already written almost all of the above, I just realized that we could apparently discern some conceptual or intensional difference between two logically true statements, such as "For all $x$, either $x = x$ or $x \neq x$" and "For all $x$, either there is or there is not such $y$ that $x = y$." Presumably, the perspective of mere logic is the conceptually least committing one, and, hence, there can be no lesser committing perspective from which to distinguish two logically equivalent first-order statements such as above. But, we seem to be able to discern their semantic difference. The vertically "compound-eye" view cannot explain this ability of ours.

I have no response to this problem, as of now."

"defining" a semantics of quantifiers in any sense of the term, but a way of merely characterizing what I have elsewhere called "double-edge theory of quantifier-use". I will write more about this theory of quantifier-use in near future for explaining it is a part of explaining what I have called "mathematese".)