How to understand the non-logical analyticity

As an example of the non-logical kind of analytic statements, Quine gives "No bachelor is married." And, in his effort to analyze this kind of analyticity, he embarks on the examination of the notion of 'synonymy,' on the basis of a presumed account of that sort of analyticity, namely, that such a non-logical statement is (nonetheless) analytic if a synonym-by-synonym replacement of terms makes it a logically true statement (just as "No bachelor is married" is turned into "No unmarried man is married"). Let me call this account of non-logical analyticity the synonym-replacement account. I think this account is wrong.¹ Let me explain how it is wrong, by providing an alternative, correct account.

The correct account is that the non-logical analyticity is essentially like the theorem-hood of an axiomatic "theory," i.e., an axiom-set. For instance, given any axiomatic definition Γ of the concept of group, that "there is at most one identity element" is a theorem. So, if there is a sense in which we can say that Γ defines the meaning of the term "identity element," then, in that sense, we can say that the said theorem is "true by definition," i.e., is a non-logical analytic statement. I assume that most people who are sufficiently familiar with axiomatic mathematics accept that there indeed is such a sense, although the notions of "definition" and "meaning" in question here may be hard to communicate to those who are not so familiar with axiomatics. So, I can see that this assumption can be controversial. For now, let me just assume this. Everything I will say in the rest will depend on this assumption.

Having said that, I think that this is the correct account of non-logical analyticity.² Let me call it the axiomatic account of non-logical analyticity, just for the sake of later reference.

Now, no matter how the term "identity element" is replaced, the theorem never turns into a logically true statement. At best, it turns into a paraphrase of a part of what Γ says. But, as Γ itself is not logically true, this theorem cannot be turned into a logically true statement. In general, if Γ is not logically true, then, it's not the case that any theorem p of Γ (i.e., a statement which is not logically true but is true-by-definition of Γ) can be turned into a logically true statement by some synonym-replacement. Of course, there is a way to make a logically true statement out of a theorem p of Γ even if Γ is not logically true: to make a conditional statement in the form of "If Γ, then p." Interestingly, by way of this Γ-conditionalization, we may be able to make a false feeling of "turning p into a logical truth by way of synonym-replacement." Consider:

- If Γ, and if x is an "identity element" iff x is [a suitable Γ-theoretic complex predication], then, there is at most one "identity."

This maneuver may give a false sense of being an example of a synonym-replacement because it involves a step of replacing a 'defined' term ("identity") by a complex term made from 'undefined' terms.

¹ In fact, I also think that Quine's inability (which is apparently shared by many of us) to see the obvious 'falsity' of this account betrays certain limit of his (and our) philosophical imagination. But, I leave this point aside for now.
² Notice that there can be an existential analytic statement by this account, as exemplified by the theorem above. Some Kant-followers may find this result as surprising. Or worse, some even may think of this as a decisive piece of evidence against this account, thinking that an adequate account of analyticity should not render any existential statement analytic. My rebuttal of such objection would require a certain new interpretation of quantification, explaining which will take extra time. If interested, please see this informal essay of mine.
of $\Gamma$. But, this maneuver clearly does not make the original statement into a logical truth just by a replacement of synonym by synonym.

"No bachelor is married" is surely a non-logical analytic statement, but that is not because it can be turned into a logical truth by replacing "bachelor" with "unmarried man." To begin with, "bachelor" is not really a synonym of "unmarried man." More carefully, "a bachelor" is a term for a sortal reference, to a "thing" qua a bearer of a much more complex and compounded property than mere "unmarried man" --- say, qua a "thing" which is a living male human-being who is already certain age and is not yet married to a female." This analysis of bachelorhood may not be free from objections (just for instance, a divorced male may be called a bachelor, though he has been married before). But, the point here is that the "meaning," or the conceptual content, of the term "bachelor," which makes the "No bachelor is married" a non-logical yet analytic statement, is to be identified by a certain "position" which this conceptual term holds in a certain system, $\Gamma$ (let me reuse this symbol), of conceptual terms, which by definition (or as a matter of definition) inter-relates such $\Gamma$-theoretical concepts and distinctions as: "human-being," "male/female" distinction, "living/dead" distinction, "human life (birth to death)," "life-stages (of a human-being)", "marriage (between an age-proper human male and an age-proper human female)," etc., etc. "No bachelor is married" is analytic only under this $\Gamma$ as it can be turned into a logical truth by way of the $\Gamma$-conditionalization by this particular $\Gamma$.

- If $\Gamma$, and if $x$ is a "bachelor" iff $x$ is a [suitable $\Gamma$-theoretic complex predication], then "No bachelor is married."

In the case of this particular statement ("No bachelor is married"), it just happens that a part of the content of the concept of "bachelor" can be concisely, but unfortunately misleadingly, taken out as "unmarried man," thereby enabling, only coincidentally, the synonym-replacement in this case. But, this is not generally the case for any non-logical analytic statement, as I explained above. If the axiomatic account of non-logical analyticity is not yet acceptable, please consider: "Anything which is red is colored;" "If it's raining now, the street will be wet later;" "If you release a piece of chalk, it will fall;" etc., etc. 3 I believe that all of these statements (usually) appear to us to be as analytic as "No bachelor is married." 4 But, there is no convenient way to make them into logical truths by mere synonym-replacement.

---

3 I think Sellars would consider all of the non-logical analytics above as cases of material inference. I think Kant's partially ingenious but partially confused notion of non-logical analyticity is properly revised and revived in Sellars's material inference. But, this is another historical issue.

4 Actually, their analyticity depends on context, which is just as it should be according to the axiomatic account. Non-logical analyticity depends on context in the sense that a statement $p$ is non-logically analytic only in a context in which a certain conceptual system $\Gamma$, which renders $p$ analytic, is assumed as the background presupposition of the context. So, even "No bachelor is married" may cease to be analytic in certain context: Imagine a polygamous society in which one is called a "bachelor" as long as one is willing to marry (more). If two people disagree about the truth/falsity of "No bachelor is married" for this reason, they may be said, in a reasonable sense, to speak different languages. The analyticity defining system $\Gamma$ is a built-in component of a language in this sense. I think this conception of language was alien to Quine (despite his holism), which constituted his "limit of philosophical imagination" (mentioned in the footnote 1).