Initial report on yin theory

Two mathematical structures, $<\mathbf{Q}^+, \div>$ (the set of positive rationals relative to division) and $<\mathbf{Z}, ->$ (the set of integers relative to subtraction) share a certain structural "form." Although these two structures are both quasigroups, the shared structural "from" is most likely more specific than that of quasigroup. In this essay, I call this hypothetical structural "from" *yin theory*,[1] to pay homage to Saussure's conception of "system of *difference* without positive terms" and to a Daoistic notion of yin-yang (negative-positive) principle.[2] (And because I don't know what it is called in mathematics.) So, I call a model of this theory a yin.

In this essay, I give a tentative definition of yin theory (Definition (Y)) and some of its basic properties (Theorems 1 - 22) derivable from this tentative definition (Y) (with a brief proof for each Theorem). (The definition (Y) may not be sufficient as a definition of yin theory. But, all the axioms in it are true of yin theory. So, all theorems derivable from it are properties of yin theory.) Then, I give a problematic Theorem 23 and its spin-off conjectures.

Reflection on the pragmatics of my own efforts to spell out and prove Theorem 23 made me re-think about the epistemology of "ordering relation." I believe that this line of re-thinking of "order" eventually leads us to a re-thinking of the foundations of mathematics, with eyes fresh and free from certain prejudices that have dominated this area of research in the twentieth century. I'm referring to the generally "constructivist" or "arithmetizing" approach that dominated the twentieth century foundations of mathematics. Yin theory will, I think, motivate a new approach, a kind of "normativist" or "axiomatizing" approach. This is also a reason why I name it yin theory: The "arithmetizing" tradition strikes me as a kind of positivist (PARTICULAR-oriented, or yang) approach, while the "axiomatizing" approach seems to be a kind of negativist (UNIVERSAL-oriented, or yin) approach.[3]

My research in this line is still in its infancy. This essay is only an initial report. Any feedback will be sincerely appreciated.

Note on a prerequisite:

My pragmatic/epistemological considerations involved in the Theorem 23 and the conjectures presuppose certain familiarity with the epistemological dualism I expressed in my previous essay, "Two ways of identification." So, at some point, I must ask readers to read it before reading on this essay. But, at least up to the presentation and the proof of Theorem 23, I will keep such presupposition minimum so that readers who haven't read the previous essay can manage to follow the discussion. My hope is that, the description and proof of Theorem 23 written in this way make

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[1] I call a structural "form" a theory because of this reason.

[2] For a more detailed explanation of this naming, see "Second outline."

[3] Two terms used here, PARTICULAR and UNIVERSAL, are put in the small caps to indicate that they are technical terms which came from my previous essay "Two ways of identification."

another entry route to the epistemological dualism of the "Two ways of identification," reaching out for mathematicians' attention.

Technical terms introduced in that essay will be marked by small caps for their first few appearances (or even later, as reminder), as is done with two terms, PARTICULAR and UNIVERSAL, used above.

Definition (Y)

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Let Y be a set and : be a binary operation defined on Y. Have <Y, :> satisfy the following axioms.

Y1. : is closed on Y.[4]
Y2. For all a, b, c, d in Y, (a:b):(c:d) = (a:c):(b:d).
Y3. For all a, b, c, d in Y, if a:b = c:d, then a:c= b:d.
Y4. There is an element e in Y such that:
- a. for all a in Y, a:e = a.
- b. for all a, b in Y, e:(a:b) = b:a.
- c. for all a in Y, if e:a = a, then a=e.[5]

[4] Previously, in "Two ways of identification," I accused mathematicians' practice of counting a closure axiom as an axiom. (I did this after introducing axioms G1 – G4 of group theory.) The accusation was correct only with respect to one aspect of a closure axiom that *delimits* the codomain of the given operation, that is, the aspect of being a sort of universal quantification: e.g., "*for all* c in U, if there are a, b in Y such that a:b = c, then c is in Y." (Here, "U" refers to the unstatable background conceptual universe of which Y is tacitly seen as a subset.) But, I was oblivious back then of the fact that a closure also stipulates an *expansion* of the codomain through an expansion of the range of the operation. That is, there is the other, hidden aspect to a closure, that of being a relationally-relativized existential quantification, e.g., "for all a, b in Y, *there is* c in Y such that c = a:b." So, I was wrong in assuming that the whole of a closure can be thrown away from an axiom set without a theoretical/contentual cost. The relativized-existential aspect should be left in the axiom set to keep the theoretical/conceptual content of the theory. But I think I was right in pointing out that the universal quantification aspect was redundant or unstatable. Here, in defining the yin theory, I leave the closure axiom in the way it usually is in mathematics only to respect the convention. But, axiomatically strictly speaking, its only significance lies in the relativized-existential quantification.

[5] (This note is intended only for those who have read "Two ways of identification.") The definition (Y) is not quite a GENUINE DESCRIPTION. Having Y4, it fails to meet the Condition (ii):

> **Condition (ii)**: In Γ, each use of existential quantification is bound by a universal quantification through a relational predicate, so that the existential claim is relativized to the universal claim (e.g., "for all x there is y such that Rxy" as opposed to "there is x such that for all y Rxy"). [Note: replacing an unbound use of existential quantification by the negation of universal one (with due modification) does not change an otherwise (ii)-failing formulation Γ into a (ii)-meeting new formulation Γ'.]

A terminological note on inverse

For all a, b, c, if a:b = c, then a:c = b, as shown in Theorem 17. This motivates us to call x:y (the result of the operation x:y) the x-*inverse* of y, treating each element x of Y as the center of a (local) inverse. (Let us use the word "inverse" ambiguously, both to refer to the symmetric binary relations determined by the centers and to the other relatum of a given relatum in such a relation.) Based on this thinking, let us use notation y^{-x} to denote the x-inverse of y, if such notation helps to clarify what is stated by the statement. This way of construing x:y is further justified by Theorem 18: for all a, b, b = a:(a:b). Using the new notation, this says: b = $(b^{-a})^{-a}$. The new notion should help to see that Theorem 18 says that, for any element a as the center of local inverse, any b is such that the a-inverse of the a-inverse of b is b, just as the inverse of the inverse of something is usually that thing itself (in the typical use of the notion of inverse).

Using this new notation, the Y4 axioms can be written as follows.

Y4-a. for all a in Y, $e^{-a} = a$
Y4-b. for all a, b in Y, $(a{:}b)^{-e} = b{:}a$.
Y4-c. for all a in Y, if $a = a^{-e}$, then a=e.

As Y4-b indicates, the element e (whose uniqueness, relative to =, is shown by Theorem 3), as a center of inverse, exhibits a unique property: For any a, b, a:b is the e-inverse of b:a. And, as Theorem 9 shows, this is a property unique to e. Besides, e also exhibits other unique properties in relation to the behaviors of :. For these reasons, we distinguish e from among all centers of inverses, and call it the center of *global* inverse of Y when it is necessary to distinguish it from other local inverses. For referential ease, let us call e simply the center of Y and the e-inverse of an element simply it's inverse, if it does not cause confusion in the given context.

Theorems

Basics:

1. : is onto. [Y4-a]
2. a:b = e iff a=b. [Y3 and Y4-a]
3. **The center e is unique.** [*Reductio*, 2, Y4-a.]
4. If a=b, then a:c = b:c and c:a = c:b. [2 twice, Y3; for each conjunct]
5. If a:c = b:c or c:a = c:b, then a=b. [Y3, 2 twice; for each disjunct] --- **A yin is a quasigroup.**
6. (a:b):c = (a:c):b. [Y4-a, Y2, Y4-a]
7. (a:b):(c:b) = a:c. [Y2, 2, Y4-a]

So, |(Y)|, i.e., the ABSTRACT STRUCTURE represented by the definition (Y), is not quite a GENUINE SIMPLE UNIVERSAL, but as close to it as, e.g., the group theory is.

8. If a:b = a (i.e., b^{-a} = a), then b=e. [Y4-a, Y3, 2, Y4-a]
9. If a:(b:c) = c:b (i.e., $(b:c)^{-a}$ = c:b), then a=e. [Y3, 6, 2, Y3, 2, Y4-a]
10. (b:a):(b:c) = c:a [Y2, 2, Y4-b]
11. If a:b = c:d, then b:a = d:c [Y3 twice]
12. (a:b):(c:d) = (d:c):(b:a). [Y4-b, Y2, 2, Y4-b]

Algebraic properties of the yin operation:

13. If a:b = b:a, then a=b. [Y4-b, Y4-c, 2] --- **not commutative**.
14. If (a:b):c = (a:c):(b:c), then c = e [7, 8] --- **does not distribute left.**
15. If a:(b:c) = (a:b):(a:c), then a = e [9, 10] --- **does not distribute right.**
16. If (a:b):c = a:(b:c), then c = e [Y3, 6, 2, Y3, 11, 6, 2, Y4-a, Y4-c] ---- **not associative**.

Local inverse: (in each of two theorems below, a serves as the center of local inverse):

17. If a:b =c, then a:c = b (i.e., if b^{-a} = c, then c^{-a} =b). [Y4-a, Y3, Y4-a]
18. b = a:(a:b) (i.e., b = $(b^{-a})^{-a}$). [Y4-a, Y2, 2, Y4-b, Y4-a]

Some characteristic properties of e:

19. a:b = a iff b=e (i.e., b^{-a} = a iff b=e). [⇐ Y4-a; ⇒ 8]
20. c:(a:b) = b:a iff c=e (i.e., $(a:b)^{-c}$ = b:a iff c=e). [⇐ Y4-b; ⇒ 9] ---- **Only e reverses order.**
21. (a:b):c = (a:c):(b:c) iff c = e. [⇐ Y4-a; ⇒14] ---- **Only e left-distributes**.
22. a:(b:c) = (a:b):(a:c) iff a = e. [⇐ 2, Y2; ⇒15] ---- **Only e right-distributes**.

Pre-yin

As shown above, a model of the definition (Y) is a quasigroup. (See Theorem 5.)[6] As of now, I'm still not sure if a model of (Y) is nothing more than a quasigroup. But, I'm more inclined to deny it than to affirm it. Notice that a proper subset of the axioms, namely, {Y1, Y3, Y4-a}, is already such that its models are quasigroups. This should be obvious from the fact that the proof of the Theorem 5 above uses only Y3 and Y4-a. So, if a (Y) model is nothing but a quasigroup, all the other axioms should be derived from this set. But, it seems difficult. Most crucially, Y2 seems impossible to derive from this set.

In fact, Y2 seems to be impossible to derive not just from {Y1, Y3, Y4-a} but even from the set of all the (Y) axioms other than Y2. However, my opinion about this should be taken with care. I only

[6] While there are a number of different ways to understand or define the notion of a quasigroup, I primarily identify it as a magma whose Cayley table is a Latin square. This is what Theorem 5 shows. (Personal acknowledgement: I thank Elliot Campbell for telling me that the definition (Y) at least fit the definition of quasigroup.)

state that, though I have made some attempts to eliminate Y2 from the definition (Y), so far, I have no success.

So, tentatively assuming that the definition (Y) cannot be reduced to {Y1, Y3, Y4-a}, I tentatively call this subset a definition of a pre-yin. (As mentioned above, a yin satisfies all the axioms of (Y).) Possibly, this set may be merely another definition of quasigroup. A pre-yin already possesses a number of properties of (Y). I list some of them below, if only for the sake of ease of reference.

1. : is onto. [Y4-a]
2. a:b = e iff a=b. [Y3 and Y4-a]
3. The center e is unique. [*Reductio*, 2, Y4-a.]
4. If a=b, then a:c = b:c and c:a = c:b. [2 twice, Y3; for each conjunct]
5. If a:c = b:c or c:a = c:b, then a=b. [Y3, 2 twice; for each disjunct] --- **quasigroup-hood**
8. If a:b = a (i.e., b^{-a} = a), then b=e. [Y4-a, Y3, 2, Y4-a]
11. If a:b = c:d, then b:a = d:c [Y3 twice]

Theorem 23

23. The relation ≤ linearly orders Y.

The binary relation ≤ will be defined below. This process of defining ≤ will be pragmatically convoluted. Thinking about the pragmatics of this process is the main content of this section.

In fact, I came up with this Theorem not by thinking about what theorems would follow from the definition (Y), but about what properties should be exhibited by yin theory, i.e., the structural "form" shared by <**Q**$^+$, ÷> and <**Z**, ->. The same is more or less true of most of the Theorems above. But, they are luckily easily expressed in the terms introduced by the definition (Y). This is not the case with the Theorem 23 because of the relation ≤. To be sure, this is still a theorem of (Y). I will prove it. But ≤ may or may not be really an integral part of the definition (Y), that is, may or may not be really "definable" solely using the primitive terms of (Y). A further complication is that, if something extra-(Y) sneaks in during this process, then, what sneaks in may or may not be such that we can fix the problem by way of extending (Y) by adding more axiom to it. Details of this complication will be discussed after ≤ is defined and the Theorem is proved. But, readers are asked to pay close meta-pragmatic attention to the pragmatics of the process of defining ≤.

Let $\equiv_e$ be an equivalence relation on Y such that:

i) for all a, b, if a:b ≠ b:a, then a:b $\not\equiv_e$ b:a,
ii) for all a, b, c, if a:b $\equiv_e$ a:c, then b:a $\equiv_e$ c:a,
iii) for all a, b, c, if a:c ≠ c:a, then (if a:b $\not\equiv_e$ a:c, then a:b $\equiv_e$ c:a), and
iv) for all a, b, c, if a:b $\equiv_e$ b:c, then a:b $\equiv_e$ a:c.

(I used the subscript e to indicate that we can define a similar kind of equivalence relation not just relative to e, but relative to any local center of inverse, and that any such equivalence relation will do for our purpose of defining the relation ≤.[7] From here on, for ease of denotation, let us drop the subscript.)

Among the above conditions for ≡, (i), (ii), and (iii) form one natural group, in distinction from (iv). For one thing, (i) through (iii) together work to achieve one thing, and (iv) adds something distinct to the result of (i) – (iii). To clarify this, let me "translate" the conditions (i) - (iii) for what they "mean."

i) x and its inverse are not ≡ equivalent, unless x = e,
ii) if x and y are ≡ equivalent, then so are their respective inverses,
iii) if x is not ≡ equivalent to y, then x is ≡ equivalent to y's inverse, unless y=e.

(For a hint of why the original (i) – (iii) can be reformulated this way, see the footnote 7.)

Following the ACQUAINTANCE-EPISTEMOLOGICAL side of our mathematical intuition, we can and must say that any equivalence relation on Y partitions the set Y. (Note that the notion of "partitioning a set" is acquaintance-epistemological insofar as it treats the set as a COMPLEX PARTICULAR consisting of SIMPLE PARTICULARS, i.e., its elements.) The conditions (i) through (iii) makes this partition a partition into *three* equivalence classes, say, A, B, and {e} (unless Y is {e}) such that for any x and y, $x, y \in A$ iff $x^{-e}, y^{-e} \in B$.[8] And, they seem to do nothing more than that.[9] So, following the acquaintance-epistemological side of our intuition further, especially led by its characteristic *combinatorial* thinking, we naturally come to wonder whether, with the conditions only up to (iii),

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[7] Note that the first three conditions for $\equiv_e$ can be stated in the following way:
(i) for each x, if $x \neq x^{-e}$, then $x \not\equiv_e x^{-e}$,
(ii) for all x, y, if $x \equiv_e y$, then $x^{-e} \equiv_e y^{-e}$,
(iii) for all x, y, if $y \neq y^{-e}$, then (if $x \not\equiv_e y$, then $x \equiv_e y^{-e}$).
(We can reformulate these conditions in this way because a (Y) model is a quasigroup, that is, because of Theorem 5. I will explain this more fully in a later section.) Now, we can define an equivalence relation $\equiv_a$ for any a, up to the condition (iii), by defining it such that:
(i) for each x, if $x \neq x^{-a}$, then $x \not\equiv_a x^{-a}$,
(ii) for all x, y, if $x \equiv_a y$, then $x^{-a} \equiv_a y^{-a}$,
(iii) for any x, y, if $y \neq y^{-a}$, then (if $x \not\equiv_a y$, then $x \equiv_a y^{-a}$).
For the condition (iv), we can simply add:
(iv) for all x, y, z, if $x{:}y \equiv_a y{:}z$, then $x{:}y \equiv_a x{:}z$.

[8] Proof: First, the equivalence class containing e contains no other element because: Suppose it did contain any element that is not e, say, a (i.e., a ≡ e). By Y4-a, a:e ≡ e:e. By (ii), e:a ≡ e:e. Hence, a:e ≡ e:a which contradicts (i) (because a:e ≠ e if a≠e, by Theorem 2). Second, if there is an equivalence class A other than {e}, then there is at least one more such class because: for each element a of A, there is the element e:a, that is, a^{-e} (because of the closure of : on Y), where $a^{-e} \not\equiv e$ (by the above) while $a \not\equiv a^{-e}$ (by (i)). Third, there are no more than three equivalence classes because of (iii). Let me omit the detail. Let me also omit the details of the proof that: for any x and y, $x, y \in A$ iff $x^{-e}, y^{-e} \in B$. They should be obvious. (I also omit the proof for the analogous claim for $\equiv_a$ --- see the footnote 10 below for the detail of the "analogous claim.")

[9] I have no proof for this claim as of now. It is claimed only on an intuitive basis.

there may be more than one such equivalence relation ≡, that is, whether there may be more than one way to partition the set Y in accordance with these conditions, if the set Y in question is large enough. What the condition (iv) assures is that there is only one way to tri-partition Y, if (iv) is required on top of the (i) – (iii). (I postpone a full proof of this until due time. For now, suffice it to say that (iv) assures that the relation ≤, which is defined in reference to the ≡ satisfying (iv), will be transitive.)

It is in this sense that (iv) is distinct from (i) – (iii), *for one thing*. Put differently, the conditions (i) – (iii) seem to be distinct in that they do not appear to actively interfere with the way the definition (Y) arranges the elements of Y into the theoretically stipulated structural "form" but appears only to passively pigeonhole them into classes according to these equivalence conditions. By contrast, the condition (iv), in allowing only one way of such partition, appears to actively interfere with the original arrangement of Y, that is, to require Y (a set, i.e., a COMPLEX PARTICULAR, which is to be thus partitioned by ≡) to be more than just a (Y) model. (This appears so unless (iv) is already entailed by (i) – (iii) on the basis of (Y) --- see the next paragraph.) Put differently yet, it seems that the equivalence relation ≡ which is defined only by (i) – (iii) seems to be a purely *descriptive* tool, while adding (iv) to the definition seems to make it partly a *normative* tool, tacitly narrowing down the range of sets we are talking about.

The condition (iv) might be distinct from (i) – (iii) in yet another sense. Although I have not found a proof, I am wondering if (iv) might follow from (i) – (iii), together with the axioms of the definition (Y). If we can derive (iv) from (i) – (iii) and (Y), then we do not need (iv) to define ≡; that is to say, the equivalence relation defined only up to (iii) is sufficient to tri-partition Y in a unique, unambiguous manner. [10] (Of course, the uniqueness in question here is only up to =.)[11] But, as of now, I have no proof for (iv) from (i) – (iii) and (Y).

Now, suppose that (iv) does not follow from (i) – (iii) and (Y). This entails that there are more than one equivalence relation that satisfies (i) – (iii) (relative to =).[12] (Let us denote such an equivalence relation by $\equiv_{(i)\text{-}(iii)}$ for a while.) Even in that case, it may be that we *can choose* or *construct* one

[10] Similar things can be said of any $\equiv_a$. Let √a denote such b that a:b = b, if such b exist in Y, and denote nothing if such b does not exist. Then, first, any relation $\equiv_a$ partitions Y into three or two equivalence classes (depending whether √a exists in Y or not), say, A, B, and {√a} (or A and B), such that for any x and y, x, y ∈ A iff x^{-a}, y^{-a} ∈ B. (I omit the proof.) Secondly, there may be more than one such equivalence relation (insofar as there may be more than one way of this partition). Finally, the definition (Y) may or may not allow more than one such partition.

[11] Note --- or recall from the "Two ways of identification" --- that in an axiomatic theory, even what appears to be acquaintance-based identity of particulars is in fact only description-based theoretical "identity" which is only relative to or up to the theoretical "indiscernibility" of =. I'm saying that relative to this theoretical "identity" (and we are only concerned with this sort of "identity" in purely theoretical discourses on a given axiomatic theory), there may be only *one* way to partition Y into A, B, and {e} to the satisfaction of (i) - (iii).

[12] For a proof, suppose that (iv) does not follow from (i) – (iii), and yet (for *reductio*) that the yin theory allows only one such equivalence relation, $\equiv_{(i)\text{-}(iii)}$). Then, no equivalence relation $\equiv_{(i)\text{-}(iii)}$ satisfies (iv). But, a tri-partition of $\mathbf{Q}^+$ into {a∈$\mathbf{Q}^+$| a<1}, {1}, {a∈$\mathbf{Q}^+$| 1<a} defines an example of one equivalence relation $\equiv_{(i)\text{-}(iii)}$ that satisfies (iv).

particular $\equiv_{(i)-(iii)}$ from among many, in such a way that the chosen/constructed one happens to satisfy (iv) (in addition to (i) – (iii)) --- let us denote it by $\equiv_{(i)-(iv)}$. If we assume that we *can* choose/construct such one, on that assumption, we would be allowed to refer to (and, so, talk about) *the* equivalence relation $\equiv$ (in the sense of $\equiv_{(i)-(iv)}$) with the definite particle, even if we *have not actually chosen/constructed* one. This is especially so because there is only one such equivalence relation that satisfies (iv) (although its proof is postponed).[13]

As many readers might have realized by now, the assumption, that we can choose/construct one $\equiv_{(i)-(iii)}$ (one that is $\equiv_{(i)-(iv)}$) from among many, suspiciously smacks of the Axiom-of-choice flavor. This could well be a reason to think that the yin theory (and, so, the definition (Y)) after all allows multiple ways of the $\equiv_{(i)-(iii)}$-satisfying partition and we after all have to make this choice-assumption to uniquely identify, that is, pretend to uniquely identity, one $\equiv_{(i)-(iii)}$ that is $\equiv_{(i)-(iv)}$. For, my purpose in trying to introduce the single equivalence relation $\equiv_{(i)-(iv)}$ is eventually to prove that a yin is linearly ordered, that is, well-ordered (without orientation --- see below) by a certain binary relation that is definable in reference to *that* $\equiv_{(i)-(iv)}$. It is said that the Axiom of choice and the well-ordering theorem are equivalent. It makes sense if an effort to prove that a yin is well-ordered turns out to require an assumption which is equivalent of the Axiom of choice. Yet, my willingness to conjecture that a yin is linear-ordered *without orientation* (see the Conjecture (P) below) makes me think that yin theory, if defined well, might after all allow a proof of (iv) from (i) – (iii), i.e., allow only one equivalence relation $\equiv_{(i)-(iii)}$ which by theoretical necessity (i.e., by stipulation of the theory) cannot fail to be $\equiv_{(i)-(iv)}$. As of now, I'm inclined to conjecture that way. That is, I think that either the definition (Y) allows the proof of (iv) from (i) – (iii), or, at least, (Y) can be extended to (Y)+ which allows this proof. But, to repeat, I have no proof yet. I do not even know if (Y) allows or does not allow such a proof.

Our metapragmatic reflections have by now been rather complicated. Let me wrap up what we will have to be assuming in our future *definite* reference to the equivalence relation $\equiv$, *if we continue our discussion on the basis of the definition (Y).* (Let us drop the subscript from such a definite reference to $\equiv$, from now on, understanding that such a definite reference always refers to $\equiv_{(i)-(iv)}$). It is either that:

(a) the definition (Y) allows only one relation $\equiv_{(i)-(iii)}$ (which is *by theoretical necessity* $\equiv_{(i)-(iv)}$); so, our definite reference to the $\equiv$ is a *theoretical* reference (enabled by IDENTIFICATION BY DESCRIPTION),

or that:

[13] For those who have read the "Two ways of identification": Note that such a *choosing* is an IDENTIFICATION-BY-(INDIRECT) ACQUAINTANCE, that is, a *construction*, of a complex particular from simple particulars. This is because the possibility of the multiple ways to satisfy the conditions (i) – (iii) is brought to our consciousness by the acquaintance-epistemological side of our mathematical intuition. (The possibility in question is a *combinatorial* possibility.)

(b) the definition (Y) allows more than one such relation $\equiv_{(i)\text{-}(iii)}$, but we can "choose" one from them (such that the chosen one is *in fact* $\equiv_{(i)\text{-}(iv)}$), and we have "chosen" one that way; so, our definite reference is *direct* reference (enabled by IDENTIFICATION BY (INDIRECT) ACQUAINTANCE).

If what we will be assuming is (b), then we will have shifted, or will be pretending, by making that assumption, as if we have shifted, to the acquaintance-epistemological mathematics, parting with the purely description-epistemological axiomatic mathematics started by the definition (Y). (That is, in this case, we are from now on to verbally pretend *as if* we have *constructed* $\equiv$, although we haven't.) So, to re-phrase the above summary of (b) more carefully, our definite reference to the relation $\equiv$ is a *hypothetical* direct reference, hypothetically made possible by the hypothetical initial baptism of the $\equiv$ which accompanied the hypothetical construction (choosing) of the $\equiv$.[14] If we are assuming (a) rather than (b), we have not made such an epistemological shift or pretension of shift. We don't know which is the case yet. In any case, let us start to *definitely* refer to, and talk about, *the* equivalence relation $\equiv$.

Now, whether we are assuming (a) or (b) makes a difference for what we must be assuming in referring to --- or more precisely, *directly* referring to --- one or the other of the two equivalence classes (those two other than {e}), by calling it, say, A. If we are assuming (a), our direct reference to A renders us assuming that we have performed an initial baptism for A, and have made it possible for us to *directly* refer to it through this baptism. This is because, insofar as the definition (Y) is concerned, there is no way to distinguish between the two equivalence classes. So, in this case, our direct reference to A is rendered a moment of relaying the historical chain of reference which is initiated by *that* baptism. Having made this explicit, it should be clear that, in this case, the identity of A (or what our symbol "A" is supposed to refer to) is relative to "us" *qua* the carriers of *that* historical chain of reference going back to the naming of that class as A. ("We" are, in *this* case, *not* the carriers of the chain of reference going back to the *initial* baptism of one among many $\equiv_{(i)\text{-}(iii)}$'s as *the* $\equiv$, because "we" haven't performed such baptism.)

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[14] "Initial baptism" is a term and a notion advocated by a philosopher Saul Kripke (in his "Naming and necessity"), together with a notion which is by now known as "historical chain of reference." (The latter notion is also known as "causal chain of reference" or "causal-historical chain of reference." However, the term "causal," though used by Kripke himself, is unnecessarily misleading, in my opinion. In this essay, I will avoid using "causal" in referring to this notion.) Roughly, an initial baptism is a speech act of giving a proper name to something, so that the speech community from then on can *directly* refer to it by using the name, either mediately (through intervening linguistic devices such as pronouns) or immediately (by using the name). (The notion of *direct reference* is contrasted against *reference by way of description*, that is, by way of specifying sufficient properties to identify the referent.) Once a name is introduced in this way into a language, a speech community apparently behaves in such a way that the name's "power" to directly refer to the referent remains with the name, as if moments of use of the name relay a chain of reference (started by the initial baptism) from one link to another. Obviously, mathematical discourses teem with such baptisms and direct-referential chains, as much as they teem with references by description. A mathematical discourse is a micro evolution of language, indeed.

By contrast, if we are assuming (b) and assuming that we have *directly* referred to the ≡ when we definitely refer to it, we have no need to assume that we have made this extra *initial* baptism. We have been acting all along as if we had constructed the ≡ and baptized it on that acquaintance-basis, in this case. So, A and B are directly acquainted by us all along, in this pretension. So, our direct reference to A is rendered a moment of relaying this pretended chain of reference going back to the pretended initial baptism for the ≡.

Either way, we can refer to A only as a direct reference (or as a pretended direct reference), made possible by an initial baptism performed (or pretended to have been performed) at one point or another in our discourse history (or pretended discourse history). Having made this clear, let us allow to directly refer, or pretend to directly refer, to A.

Next, let us finally define the binary relation ≤ in reference to *this* A:

Definition of ≤: Let a and b any element of Y. a≤b iff b:a ∊ A or a=b.

Now, carrying the historical chain of reference of whichever kind, we are finally ready to state Theorem23, which I repeat here for convenience.

23. The relation ≤ linearly orders Y.

For convenience, I recite the definition of a linear ordering (well-ordering) relation as well.

A binary relation R defined on a domain <M,=> is a linear order relative to <M,=> if it is such that:[15]

a) for all a, b in M, aRb or bRa (totality),
b) for all a, b in M, if aRb and bRa, then a=b (anti-symmetry), and
c) for all a, b, c in M, if aRb and bRc, then aRc (transitivity).

Proof of Theorem 23:

(a) If a=b, then a≤b (from the definition of ≤). If a≠b, then a:b ≠ b:a = $(a{:}b)^{-e}$ (Theorem 13 and Y4-b). Let c be any element in A. If c= c^{-e}, then c= e (Theorem 13 & 2). Since c≠ e, c≠ c^{-e}. So, if b:a ≢ c, then a:b ≡ c (Def. of ≡, clause (iii)). Therefore, either a:b ∊ A or b:a ∊ A.

(b) Suppose a≤b and b≤a. On this supposition, further suppose a≠b (for *reductio*). b:a ∊ A and a:b ∊ A (Def. of ≤). On the other hand, a:b ≠ b:a (Theorem 13). These contradict with the definition of ≡ (clause (iii)).

[15] By <M,=>, I refer to the totality and the individuality of a domain, presented in "our" *conceptual intuition*, so to speak, as the domain of "our" definition of the relation R. (The italicized phrase is a critical allusion to Kant's influential distinction between concept and intuition.) That is, the universal quantifications in the definition of R are restricted to M, as regards the totality, and relative to or up to the theoretical-conceptual "indiscernibility" =, as regards the individuality. That is, the elements of M are individuated, by the theory implicitly assumed in the discourse, relative to or up to =, as "we" define R.

(c) Suppose a≤b and b≤c. Suppose a≠b and b≠c. (If either identity obtains, obviously a≤c.) b:a ∈ A and c:b ∈ A (Def. of ≤). c:b ≡ b:a. Therefore, c:b ≡ c:a (clause (iv)). c:a ∈ A.

Conjecture (P)

Conjecture (P): There is a purely theoretical linear order that is orientation-neutral.[16]

This conjecture is the existential generalization of my belief about the yin theory (the shared structural "form" of <$\mathbf{Q}^+$, ÷> and <$\mathbf{Z}$, ->), namely, a belief that it should exhibit the property of being linearly ordered (while it is a pure theory) but orientation-neutrally. This is the property which led me to the formulation of Theorem 23.

To explain what I conjecture here (and what I believe about yin theory in this regard) exactly, the very first thing is to explain the notion of a pure theory. For this end, I must first remind readers of something from the "Two ways of identification." (The present essay strictly requires and presupposes familiarity with this essay, from now on.) It is the distinction between a formulation of a theory and an abstract structure to be defined by such a formulation:

> [A] formulation Γ of a mathematical theory is said to define (by genuine description or non-genuine description) a mathematical [abstract] structure |Γ| (a genuine simple universal or a non-genuine simple universal).

This distinction clarifies that our meta-mathematical use of the term "theory" is often ambiguous, meaning either a formulation Γ (if, in using the term "theory," our attention is more on its aspect of being a linguistic formulation) or an abstract structure |Γ| (if, using this term, our attention is more on its aspect of being a representation of the invariant structural "form" shared by all of its models --- which is also shared by all the equivalent formulations). The exact explanation of the Conjecture (P) requires this distinction. From here on, therefore, we start to explicitly use the term "theory" only to refer to an invariant structural "form" |Γ|, i.e., an abstract sense of a structure, equally defined by each of equivalent formulations Γ of that theory |Γ|. And, the symbol "Γ" will be used below for a formulation of a theory, not a theory itself. (Note that, in mathematical logic, the symbol "Γ" is often used for a set of statements of well-formed formulas, together with the term "theory." We follow this convention as regards the use of "Γ," and depart from it as regards the use of the term "theory.")

Next, also reminding readers of the definition of a GENUINE DESCRIPTION Γ OF A THEORY |Γ|, let me define a pure theory |Γ| as follows: a theory |Γ| is called a pure theory iff it has a formulation Γ that meets the Conditions (i) and (iii) of a genuine description. (A formulation Γ that meets all three

[16] This is called Conjecture (P) to indicate that it asserts the existence of something. An existential statement is a statement of a possibility. I will write about this in future.

Conditions, (i), (ii), and (iii) is called a genuine description, and a theory |Γ| that *can* be defined by a genuine description is called a genuine simple universal, if you remember.)

Condition (i): Γ contains no individual constant symbol.
Condition (iii): Γ contains no binary predicate that *has to be* interpreted as the "identity" (or "equality") in order for Γ to be a formulation of |Γ|.

On this basis, let us define the notion of a purely theoretical "thing" R: Let R be a relation, a monadic property, or an "individual object" (all relative to =). R is purely theoretical iff there is a pure theory |Γ| such that there is a formulation Γ of it (genuine description or not) which either contains R in it as a "primitive" term or allows a theorem which enables to define R as a "defined" term.

Let us say that such R is integrated in the theory |Γ|. (So, to say that R is purely theoretical is to say that there is a pure theory |Γ| which integrates R in it.)

Now, before moving on to the explanation of "orientation neutrality," recall my effort to explain Theorem 23. I started with the definition (Y). In hindsight, we can say that whatever structure defined by it was a pure theory. (The definition (Y) meets the Conditions (i) and (iii).) Then I ended up defining the binary relation ≤, at the end of a complicated meta-pragmatic self-reflection on the pragmatics of the process of defining it. This relation clearly does not occur in the definition (Y). On the other hand, what I try to conjecture by the Conjecture (P) is in part an existential generalization of Theorem 23. That is, I think that the relation ≤ is purely theoretical --- assuming that it is orientation-neutrally re-Abstracted. (This last qualification will be discussed shortly. For now (from now until then), assume that my reference to ≤ is reference to it as the orientation-neutrally re-Abstracted ordering relation.)

Now, recall the assumptions (a) and (b), concerning our definite reference to the equivalence relation ≡:

(a) the definition (Y) allows only one relation $\equiv_{(i)\text{-}(iii)}$ (which is *by theoretical necessity* $\equiv_{(i)\text{-}(iv)}$), so our definite reference to the ≡ is a *theoretical* reference (enabled by IDENTIFICATION BY DESCRIPTION),
(b) the definition (Y) allows more than one such relation $\equiv_{(i)\text{-}(iii)}$, but we can "choose" one from them (such that the chosen one is *in fact* $\equiv_{(i)\text{-}(iv)}$), and we have "chosen" one that way, so our definite reference is *direct* reference (enabled by IDENTIFICATION BY (INDIRECT) ACQUAINTANCE).

If the assumption (a) is the case, then ≤ would be purely theoretical. In this case, ≤ would be integrated in the theory |(Y)| defined by the definition (Y). However, even if the assumption (b) proves to be the case, I do not think that this necessarily means that ≤ is not purely theoretical. It all depends on the epistemological nature of what we *have to do* to allow us the definite reference to the ≡ (i.e., $\equiv_{(i)\text{-}(iv)}$).

If (b) is the case, ≤ cannot be integrated in the theory |(Y)|, for sure. So, in that case, ≤ would look to "us" as if it were not purely theoretical, insofar as "we" stay in a context in which "we" are bound only by the definition (Y). But, the definition of a purely theoretical "thing" renders this notion a metaphysical notion. Its *definiens* makes an unrestricted existential quantification: "... iff there is a pure theory |Γ| such that ...." This means that whether some "thing" R (here, assume that R is defined at least in part on a basis of a certain pure theory) is a purely theoretical is a metaphysical issue, not relative to "us" as historical-discursive agents who are identified relative to a certain discursive-norm sets of a given context.[17] As hinted above, even if (b) is the case, a possibility remains that (Y) may be extended into (Y)+ such that the theory |(Y)+| integrates ≤ in it. If We (a sort of pragmatics-free Platonic Agents) can find such (Y)+, then ≤ is still purely theoretical, regardless of how it looks to "us" (pragmatically-bound/identified discursive-historical agents).

Let me reveal my tacit assumption in thinking in that way. What makes the case of (b) problematic for our (or, at least my) hope to prove that ≤ is purely theoretical is that in that case, our process of identifying ≤ actually involves a moment of "choosing" or "constructing" some "thing" on its way to the identification of ≤. So, the question boils down to whether it is possible --- in some metaphysical sense --- to remove all such "constructive choice" moments from the process of the definition or identification of ≤ (which started with a definition of the pure theory |(Y)|). So, in thinking in the aforementioned way, I tacitly assume the following equivalence:

> To define ≤, We (Platonic Agents) have to make such a "constructive choice"
>
> iff
>
> there is no extension (Y)+ of (Y) (or of an equivalent of (Y)) which defines a pure theory |(Y)+| and which integrates ≤ in it.

In assuming this equivalence, I again make a fundamental epistemological distinction between (i) extending a pure theory |Γ| into a more specific pure theory |Γ+| (which is actually carried out by adding more axioms, or stipulations, to a definition Γ of |Γ|), and (ii) making a "constructive choice." Some may object that a moment of such an axiom-stipulation is a moment of an "constructive choice-making" enough. Given the contemporary epistemological confusion about two kinds of inferential independence, this objection is understandable. But it is based on that confusion. My response to this objection, however, takes a lot of space, and I must postpone the task to my future essay, "Two kinds of logic" (which I'm in the middle of writing, as of now, writing this revision of "Order without orientation").

To wrap up one aspect of the relation between the Conjecture (P) and the Theorem 23: To prove one aspect of the Conjecture (that there is a purely theoretical linear order) by way of the Theorem, it is enough to prove either that the assumption (a) is the case, or that the definition (Y) can be extended into (Y)+ such that (Y)+ allows only one equivalence relation $\equiv_{(i)\text{-}(iii)}$ which is by theoretical necessity also $\equiv_{(i)\text{-}(iv)}$. At least, that is what I believe. I eventually must show that I'm

[17] I have a mixed feeling about making references to such metaphysical notions in a part of my re-thinking of "mind." This might be ultimately inevitable or unavoidable. Still, I might have to come back later and re-think further about this notion. For now, I just let go of this worry, and move on.

right in believing this way, either by actually proving (a), or by extending (Y) to (Y)+ (without losing the pure-theory status) in such a way that (a) is true relative to (Y)+.

Needless to say, the above is only one aspect of the relation between the Conjecture (P) and the Theorem 23, concerning the notion of "purely theoretical" only on the assumption that the ordering relation ≤ is orientation-neutrally re-Abstracted.

Next, "orientation-neutrality." Suppose that a certain [conceptual-intuitive domain, <M, =>,](#) conceptualized under the normative commitments and entitlements stipulated by a formulation Γ of a pure theory |Γ|, is, *qua* a model of |Γ|, inherently ordered (partially or linearly). (The lengthy qualification on <M, => is because we often conceptualize such a theoretically given domain also under influence of some readily available intuitive interpretation. For instance, when we are studying a field purely as a theoretically defined abstract structure, we still often make heuristic use of an interpretation of it as a real number system.) Put differently, suppose that an ordering relation R exists in <M, => *qua* a model of |Γ|, as *stipulated* by Γ to be so. Now, if we conceptualize <M, => in this way, such a conception presupposes a sort of bipolarity in this conceptual background, again, as stipulated by Γ to be so. But, this background bipolarity does not necessarily presuppose an inherent *orientation* as to which "direction" of the order is which. That is, the theoretically stipulated bipolarity of <M, =>, which should be there if <M, => *qua* a model of |Γ| is inherently ordered, may well be that of bipolar *symmetry*. Put in a yet different way, the inherent existence of an ordering relation R in <M, => *qua* a model of |Γ| does not necessarily entail that the theory |Γ| stipulates a purely theoretical *break* of the background bipolar symmetry too (so that the two symmetric "directions" can be discerned purely theoretically). If |Γ| stipulates only an order but no such a break of symmetry, we say that the stipulated order R is orientation-neutral.

Now, some readers may find it spurious to say or think that an *ordering* relation can exist with absolutely no break of symmetry. That intuition is understandable, or, strictly speaking, correct. Here, we are involved in a sort of pragmatic complication similar to the one we are involved when asserting the non-existence of the referent of a definite reference. Our notion of order clearly *presupposes* a break of bipolar symmetry in our conceptual background. What I stated in the preceding paragraph is only an initial illustration that needs to be qualified to do justice to this pragmatic complication. Let us now turn to this qualification.

My best strategy for this qualification seems to be a way of example. The notion of an orientation-neutral order (which is purely theoretical) is perfectly exemplified (for the case of partial order) by a Boolean algebra <B, => and its Boolean partial order ⊆. Recall from the "[Two ways of identification](#)" how this theory is formulated in the second formulation. From this formulation (and assuming that B is not ∅), we can prove the unique existence of an element 0 such that for all c in B, $c \cap c' = 0$. Then, thanks to the axioms B3 and B4 together, we can prove the following theorem:

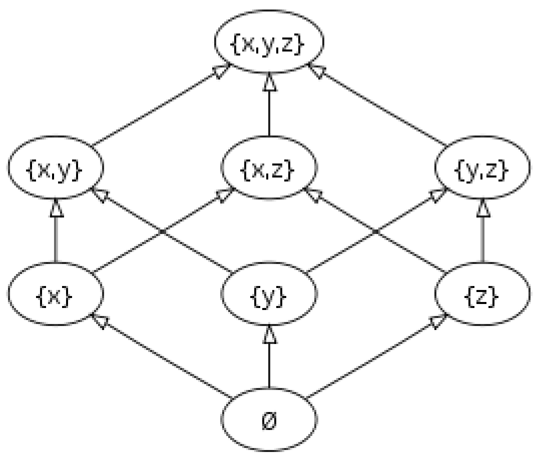
For all a and b in B, $a \cap b' = 0$ iff $a \cap b = a$.

Then, we can "define" the binary relation ⊆ so that "$a \subseteq b$" is merely a shorthand for "$a \cap b = a$." Thus, the "definition" of ⊆ would be:

$a \subseteq b$ iff $a \cap b' = 0$ (where a and b are any elements of B).

In this way, the binary relation $\subseteq$ is integrated in the theory of Boolean algebra, which is a pure theory. I just called this binary relation the "Boolean partial order $\subseteq$," for this relation is known to partially order <B, =>. (I omit a proof.) So, it is a purely theoretical partial order.

Now, that this relation does not break the bipolar symmetry of <B, =>, at least in some sense, is vividly visualized by the so-called Hasse diagram of a finite Boolean algebra.



Left: The Hasse diagram of a "three-dimensional" Boolean algebra, <P{x, y, z}, ∩, ′> (see below for "dimension")

I copied this picture from a Wikipedia page (http://en.wikipedia.org/wiki/Hasse_diagram).

The above is the diagram for a Boolean algebra with eight elements, <P{x, y, z}, ∩, ′>. Eight nodes represent the elements of P{x, y, z}, the power set of {x, y, z}, and the distribution of arrows represents the relation $\subseteq$, such that there is a chain of arrows from a to b iff $a \subseteq b$. In general, a finite Boolean algebra (i.e., a finite model of this theory), excluding the null Boolean algebra, <Ø, => (which is usually not even counted as a Boolean algebra, but I count it in), has 2^n elements where n is a positive integer. If we say that such a Boolean algebra is *n-dimensional*, then the "form" of the Hasse diagram of an n-dimensional Boolean algebra is that of an n-dimensional "box." (Here, understand the notion of "box" dimension-neutrally so that we can call a plane figure (a square) a two-dimensional box and a solid (a cube) a three-dimensional box. Similarly, a line-segment connecting two points is a one-dimensional box. The diagram above is a two-dimensional representation of a three-dimensional box, as indicated by the caption.) Though we cannot mentally "visualize" any box greater than three-dimensional in its native dimension, abstractly speaking, there are any finite-dimensional boxes. (And, though it becomes exponentially tedious after the dimension three, we can in principle represent any finite n-dimensional box two-dimensionally.)

Now, any n-dimensional box --- call it box_n --- exhibits a kind of bipolar symmetry, in the sense that we can "flip" it around in the n-dimensional Euclidean space so that the resulting box_n completely coincides with the original box_n, with the two vertices (those which represent the 1 and 0 elements in representing the n-dimensional Boolean algebra) exchanging their places. We won't be able to tell if this "flipping" has taken place or not, unless we can either identify the "absolute" position of at least one node (any node will do), or identify the "absolute" direction of the arrows.

(The above thesis, that any box_n exhibits this sort of n-dimensional bipolar symmetry, should be mathematically proven. And, probably it has been proven already. Being a sporadic amateur mathematician, I have not proved this personally, nor have found a proof made by others. As of now, I only grasp and accept this thesis by intuition.)

What I mean above by the "absolute" position/direction may be understood as the position/direction identified relative to *the orientation of the background space*. I do not mean to say such a nonsense that there is an inherent or substantival orientation in *this* "space" which "exists" only as the conceptual backdrop of our conception of $<Box_n, =>$. It is "absolute" only in the sense that it is not "relational" position/direction (or a "relationalist" position/direction, if you will) in the sense of being identified relative to the mutual relations of the constituents of the $<box_n, =>$. (For this purpose of "absolute" orientation, the $<box_n, =>$ is useless because of its symmetry.) Needless to say, this "absolute" orientation is relative orientation when it comes to the sense that it is merely a (part of) frame of reference, that is, it is something "we" ascribe, or tacitly ascribe, to the conceptual backdrop. This tacit ascription is what I called (in the "Two ways of identification") a special kind of DIRECT ACQUAINTANCE (with a conceptual background). "We" do this tacit ascription of ("absolute") orientation whenever the very notion of "flipping" a box_n makes sense to "us," or, indeed, whenever the notion of "order" makes sense to "us." After a reflection, the "flipping" is not an "objective event" that happens in the space as it is in itself, but is a "relative event" that happens in the space *as* oriented, that is, as oriented by "us."[18] Nor is the "order" of any arrangement of things an "objective property" of the "arrangement in the space" as it is in itself, but a "relative property" that is ascribed to the "arrangement in the space," where the space is oriented by "us."

[18] In trying to understand the point made by this last sentence, please ignore the nonsense of thinking/talking about the "space as it is in itself" where the "space" in question is nothing but our conceptual backdrop. The same point applies even if we were here thinking/talking about the so-called "physical space-time." (I changed the term from "space" to "space-time" assuming that the special relativity no longer allows us to have a tenable concept of space independent of time, while the notion of *orientation* applies equally regardless of whether the object in question is conceived of as "space" or "space-time," insofar as we understand the "space-time" in the "geometrical" manner which we do.)

Indeed, in my opinion, this notion of the "physical space-time" (of which we are supposed to be able to ask a question "Is it Euclidean or not?" as an empirical question) is a return of the good old Newtonian (substantival) conception of space (and time), which had been once rendered a history by the special relativity. It seems to be somehow resurrected by a very common interpretation of none other than the *general relativity*, and I have never understood why. Either I totally misunderstand the general relativity (either the theory itself, which I am yet to study mathematically, or the interpretation in question) or something incredibly naive actually happened in the scientific/philosophical community in interpreting the general relativity theory. I shouldn't be so talkative about something I don't really understand. But, with the caution that I don't know anything about the general relativity, I still want to express my strong suspect of the latter possibility. This suspect comes from a little knowledge of a certain history of geometry (and epistemology of geometry) at about the turn of the last century. I intend to write about this someday, if I live long enough, and have fortunate enough life to study the relativity theory.

Still, *when* we understand the "flipping" or "order," we cannot at the same time have this relativist attitude to the orientation (a frame of reference). We have to treat it as if it were an "objective property" of the "space," insofar as it serves as a "frame of reference." I take this to be one of "our" fundamental pragmatic inevitabilities.[19] It is because of this, that we have to re-Abstract our conception of a purely theoretical ordering relation (whatever kind it is), in order to conceive its orientation-neutrality properly (assuming that it is in fact orientation-neutral, that is, the theory in which that relation is integrated does not break the bipolar symmetry).

So, one reason why I call the Boolean partial order "orientation-neutral" (and I have one more reason, to be explained shortly) comes from the distinction between the "absolute" and "relative" orientation in the aforementioned sense, on the one hand, and from the recognition that the "direction" of this partial order is defined only in the "relative" sense (in the sense of being relative to the mutual relations of the constituents of <box_n, =>), on the other. So, recall the definition of ⊆.

$a \subseteq b$ iff $a \cap b' = 0$ (where a and b are any elements of B).

This definition ascribes a "direction" to the relation ⊆, as shown by the reference to the element 0 in the *definiens*. But, that means that the direction here is defined relative to the distinction as to which of the two "poles" is 0 and which is 1. So, strictly speaking, the "direction" of this ordering relation constitutes only a "relative" orientation (or "relationalist" orientation, if you will).

But, the abstract structure that substantiates the "existence" of this ordering relation is symmetrical, as shown above. The abstract structure of Boolean algebra by itself does not inherently recognize the difference between 0 and 1. We can define the relation ⊆ such that the arrows in the Hassse diagram would go from 1 to 0 (as opposed to 0 to 1, as is the case in the above diagram). But, the resulting diagram is identical with the original: we can "flip" it around in the n-dimensional space, and they perfectly coincide. Thus, if we were ascribing some "absolute direction" to the ⊆ relation, it is not inherent in the theory itself. It is something extra-theoretical. And, as mentioned above, we do ascribe such "absolute" direction to the ⊆ relation, whenever we consider it an ordering relation, as a matter of pragmatic inevitability.

Besides, we also do so as soon as we semantically interpret the theory (of Boolean algebra), whether as the algebra of sets or as the algebra of propositions. We do not think that, e.g., whether to conceive of a given inferential situation as "A entails B" or "B entails A" is a mere matter of "choice," not even in a looser sense of the term, "conventional choice." In this way, our semantic intuition and our pragmatic inevitability coincide and collaborate to make us oblivious to the syntactic orientation-neutrality of the Boolean partial order.

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[19] In saying this, I'm expressing my view that our spatio-temporal perception is at least in part a discursive phenomenon or event, in an inseparable relation with our discursive normativity. I will write more on this in "Two kinds of logic."

I jumped ahead a bit, but this is the second reason why I call the Boolean partial order, construed as a purely theoretical "thing," orientation-neutral. Because of the theoretically stipulated *symmetry* of Boolean algebras, our ascription of the "absolute" direction to the order is (which our pragmatic inevitability, as well as semantic application of the theory, forces us to do) completely arbitrary, or unmotivated, or "without of loss of generality," from the purely theoretical point of view. Let us call it the (guaranteed) orthogonality of the choice to the ensuing discourse. In making such a choice, we exercise our *free will* in defining or fixing the "absolute direction" of the ordering relation and together with it, *orienting* our conceptual background, freely, at our whim. It is not "we" as bound by the discursive norms of the formulation of the theory, but "we" as entitled to the freedom beyond the jurisdiction of these norms, that ascribe the "direction" to the ordering relation and ascribe the orientation to the background conceptual domain. Because of this orthogonality, the Boolean partial order is still "orientational-neutral" in the "absolute" sense, although it is "relatively oriented."

What I mean to claim by the Conjecture (P) is that a similar situation (purely theoretical order without "absolute" orientation) occurs for the case of linear order as well, with some pure theory. If we assume either (a) about our reference to the ≡ (or the extendability of the definition (Y) into the aforementioned (Y)+), then what Theorem 23 shows is that any model Y of the theory|(Y)| (or |(Y)+|) is already linearly ordered with a "relative" orientation by some linearly ordering relation that can be a part of (Y) (or (Y)+), but its background bipolarity is still orientation-neutral in the aforementioned "absolute" sense because of the symmetry of the theory.

At this point, some readers might be puzzled about the Conjecture (P). Models of any theory whose models are linearly ordered by a theory-inherent linearly-ordering relation R, form, relative to that relation, a conspicuously (bipolar) symmetrical structure, namely, the structure of a chain. Isn't it then obvious that there are purely theoretical linear order that is orientation-neutral?

The puzzling is very natural. It all depends on whether there is a purely theoretical linear order at all. It there is, it is by necessity orientation-neutral. Here, I may be revealing my laughable ignorance about abstract mathematics. But, I personally know of no pure theory |Γ| which has a formulation Γ such that it in effect contains a binary relation R in it where this R proves to linearly order any of its models, in the way the Boolean partial order does. The Conjecture (P) conjectures the existence of such a pure theory. If such a theory is known in mathematics, I should be laughed at by making such a big gesture in making this "Conjecture." I'm ready to be laughed, and want to know which theory does this. If anyone tells me kindly of such a theory, I will honestly appreciate it.