A series of essays toward two ways of identification

Essay 1.

2/11/2011 (Last revised, 3/3/2011)

Kripke, in his "Naming and Necessity," identifies a tradition in analytic philosophy, attributing it to Frege and Russell, in which reference by signs such as linguistic expressions (more precisely, tokens thereof) is pictured to be carried out by specification of properties possessed by their referents. In this picture, even our uses of proper names such as "Saul Kripke" work in this way. Kripke criticizes this picture, and gives an alternative: the initial baptism and historical relay of (correct) referential uses of names. Details of this alternative are not important for the purpose of this essay. What's important is what seems ultimately to have motivated him to this criticism. It seems his denial of the identity of indiscernibles, the principle that assures that it is possible to identify, and, hence, refer to, a particular by enumerating its characterizing properties. But he leaves open the opposite possibility: the possibility to identify a property by enumerating its instantiating particulars. Both possibilities, if construed in a little more abstract way, are assumed in more than one context in our natural ways of thinking and talking about the world and ourselves. The second possibility (to identify a property by enumerating particulars) has met skepticism in some contexts, too. For instance, in my reading, Hume's skepticism of induction is skepticism about this abstract possibility in one context. So is Wittgenstein's skepticism of rule-following. I think it is useful to call this abstract possibility the possibility of Abstraction, broadly taken as the identification of a universal by enumeration of its instantiating particulars. I think it's useful precisely because it allows us to think of an equally abstract opposite possibility, such that what Kripke denies in "Naming and Necessity" can be seen as this possibility (in one of the contexts in which it is assumed), which stands in an important contrast to what Hume and Wittgenstein denied. It is the possibility of the identification of a particular by enumeration of its characterizing universals.

What to call this general possibility, to contrast it to its opposite, the possibility of Abstraction, is a trivial but difficult problem for me (a non-native speaker of English). I cannot think of any English word that fits this notion. Only for want of the suitable term in my vocabulary, I call it the possibility of De-abstraction. By "De-abstraction," it is technically and strictly meant the identification of a particular by enumeration of universals.

In my view, there are two fundamentally different *epistemologically correct* ways of identifying something. Universals are to be identified in one way, and particulars in the other. Explaining these two ways is the objective of this series of informal essays, of which this Essay 1 is the first. Very roughly speaking, in a visual sort of figure of speech, the proper way of identification for universals is top-down, and that for particulars is bottom-up. Readers who just read the previous sentence might wonder that there was a typo there, "top-down" and "bottom-up" being reversed. This is not a typo. But I put off its explanation until the Essay 2. What I want to claim here is that there is a fundamental epistemological gap between two realms, that of universals and that of particulars, *as objects of our epistemic act of identification*.

The Abstraction, as I use this term technically here, is an epistemologically *illicit* way of identification that pretends to work across this fundamental gap, from the realm of particulars to that of universals. The same holds for the De-abstraction, crossing the gap in the opposite direction. Please make no mistake. These two, Abstraction and De-abstraction, are epistemologically *incorrect* ways of identification, and not what I mean to explain in this series of essays (although they will be also inevitably explained along the way).

Moreover, criticizing these two assumptions, of the possibilities of Abstraction and De-abstraction, is not the main point of the present series of essays. I use these criticisms mainly for a vivid illustration of what I mean by two ways of identification. But, this distinction between the two ways, that is, the claim of the fundamental gap between the two realms of objects of identification, if accepted, has a more critical consequence for the traditional epistemology: The said fundamental gap renders it impossible, or epistemologically illicit, to genuinely refer to a particular *and* genuinely predicate of it. If one can make a genuine reference to a particular, one is in the epistemological mode (the bottom-up mode) in which one can identify particulars epistemologically properly but not universals; so, one cannot do the genuine predication. If one can do the genuine predication, one is in the other epistemological mode (the top-down mode), and, hence, cannot make the genuine reference. The ubiquitously accepted notion that there are, or can be, propositional content consisting of reference and predication should be rejected, if my fundamental gap claim (i.e., two ways of identification claim) is accepted.

This is one of the reasons why I think our natural conception of truth is --- well, for want of a better expression --- incoherent. However, the existence of "us" (as the thinking/talking beings supposedly we refer to by using first-person pronouns) depends on our committing ourselves in practice to this epistemological incoherence, as (roughly) explained in the Outline of the theme of the present webpage, re-thinking of "mind."

Essay 2.

2/14/2011 (Last revised 3/17/2011)

Explanation of the two ways of identification through examples

[The present Essay 2 was initially written as Essay 3, and the one that now given as Essay 3 was initially written as Essay 2. I switched the order and revised each Essay substantially as a result, because doing so seemed to enhance the readability of these Essays a lot.]

1. Designation

I decided to call the two *proper* ways of identification (i) identification by *description*, which is the way of identification for universals, and (ii) identification by *acquaintance*, which is the way of identification for particulars. This pair of terms is of course the one originally coined by Russell in his 1910 paper, "Knowledge by acquaintance and knowledge by description." I borrow from him both the terminology and the basic conceptual distinction itself. But, following Kripke, I reject Russell's assumption of De-abstraction, that is, of the possibility that description can individuate a particular object in terms of universal properties, working across the epistemological gap. This rejection is to be taken broadly, as including the rejection of the possibility that description can individuate a particular or singular proposition (one that purports to represent a singular event or state of affairs). A little more carefully put, what I reject is the epistemological soundness of such an individuation, but this naturally commits us (me) to rejecting the very possibility of *fact* or *reality* of such individuation. So I gloss over the difference in the present series of Essays. [For more clarification on this point, see my comment on this series of Essays, on the website.] Rejecting this possibility means (among other things) that I reject the epistemological soundness/ontological (or metaphysical) possibility of the familiar kind of "logical" analysis of the notion of *uniqueness*, which relies on the intuitive semantics of the binary predicate of identity, as used in, e.g., Russell's famous "descriptivist" analysis of "the" (of definite singular reference to a particular). [As I will clarify below in this Essay, this analysis is epistemologically acceptable only if it is understood merely as an expression of descriptive indiscernibility, so to speak, rather than as an analysis of the ontologically loaded notion of *uniqueness*, which notion we can seem to make sense only as a sort of acquaintance-wise indiscernibility, as the related phrase "*numerical* identity" suggests. All of this should become clear through this Essay.] Similarly, if Russell had any assumption that it is possible to individuate a universal (a general proposition or a property) from acquaintances with particulars (singular propositions or singular objects), I have to reject that too. This means (among other things) that I reject the possibility of defining whatever universal by means of enumeration of its extension --- a ubiquitous practice in logic and mathematics. (Just in case: Rejecting this possibility is not to be understood in a way in which many people might understand Hume's induction skepticism, a claim of *impossibility* of induction, as the "impossibility in principle" (as opposed to "impossibility in practice," whatever this contrast is supposed to mean) of affirming the truth of a genuine statement of a (hypothetical) law through experiences (without resorting to a circular argument). In this understanding of Hume's induction skepticism, a certain conception of the relation between a "law of nature" and a "singular fact" is completely presupposed, together, inseparably, with the completely presupposed conception of "truth." My rejection is to deny a kind of *coherence* of that assumed conception of the relation between a universal and a particular, together with that conception of "truth.")

[Note: My attempt to reject the epistemological soundness/ontological possibility of those (apparent) epistemic acts of ours --- identification of a particular by the so-called definite description and identification of a universal by means of enumeration of its "extension" --- is *not* to imply that we should or can reject our natural conceptions of reference and predication, according to which these illicit ways of identification are not only possible but actually carried out by us all the time. Of course, in a sense I mean to say that we, as philosophers, should at least try to do so. And, my attempt for the re-thinking of "mind" is even an attempt to develop an alternative picture (involving a re-thinking of the traditional distinction between the descriptive and the normative). However, such is in a sense an impossible task for "us." In my view, "we" cannot get out of these "incoherent" conceptions of reference and predication (things we do all the time) without instantiating a contradiction between what we say and what we do (in *saying* it).]

What I mean by this pair of notions --- identification by description and identification by acquaintance --- is probably best explained through examples. Also, if, for some people, my conceptions of "particulars" and "universals," treated as two kinds of *objects of identification,* have been as elusive as the notion of two ways of identification, the same examples should also help them to clarify their puzzlement. Let us move on to this.

[Note: what is implied by this epistemological conception of the universal/particular distinction is *not* the ordinary kind of nominalism or non-realism. What it implicitly challenges is not the reality of universals (where such a challenge often assumes unobjectionable reality of particulars), but the reality of the distinction itself --- that is, the conception of this distinction *as* a distinction between kinds of *beings* as they are in themselves, and, so, *as* a distinction independent from the subject ("mind") to whom those "beings" *stand as* a universal or *as* a particular. Admittedly, this challenge still leaves room for the intelligibility/coherence of such notion of reality, or the ontological independence of the object (things to be identified) from the epistemic subject (the "mind" that does the identification). Some may even think that my challenge not only leaves room but in fact relies on it. They are correct in thinking so. Nonetheless, this challenge will be further extended, in a larger context of my re-thinking of "mind," into all sorts of conceptual distinction. In particular, the conception of the objective/subjective distinction as an objective distinction, independent from the subject to whom things (judgments, proposition, etc.) stand as objective or as subjective, will be challenged --- as long as this distinction *can* concerns "us" at all. So, if my philosophical theme is labeled as a version of non-realism, it should be noted in parenthesis that what it claims is not the non-reality of this or that traditionally accepted notion (such as universals, space, time, laws of nature, the physical universe, etc., or mind, secondary quality, qualia, etc.), but the non-reality of the notion of reality itself. Of course, stated this way, this thesis (or "utterance" of it) is an unmistakable example of the kind of self-contradiction between what it does and what it says. But this seems just how it should be, according to what it says (or, will say when it is fully stated). But, these further complications do not come in this series of essays yet. In this series of essays, my objective remains within the explication of the epistemological conception of the universal/particular distinction, and some of its immediate implications.]

2. Example of identification by acquaintance

Now, our example. Suppose that we are instructed (say, in a mathematics textbook) as follows: Let a and b be distinct objects and S be the set {a, b}. If we are instructed in this way, we are instructed to take each of a and b as a particular that is identified (and identifiable) *directly*, without further ado. It is as if we are to take them, or more precisely, referents of these symbols, as something *directly presented* to us, or to our intuition or perception (*Anschauung*), so that we can treat "a" and "b" as what Russell called "logically proper names." It is primarily this kind of "intuitionistic" identification that I call the identification by acquaintance, and objects of this kind of identification that I call particulars. But this is not all. If we are instructed in the aforementioned way, we are also to take it for granted that we can use them as *undefinable definers* so that things may be defined (i.e., identified) in terms of them (just as S is already defined in that way, in the instruction). Of course, they themselves are in absolutely no need of definition (or in no position to be so defined) during the discourse because they are directly *given* to us. Thus, for example, we will understand what an ordered-pair, <a,b>, is in terms of a and b, as an ordered-pair of *these* undefinable definers (as long as the instruction is effective). Similarly, we will understand all the subsets of S, i.e., ∅, {a}, {b}, and {a,b}=S, as defined in terms of them. If they are so understood (as each to be identified in terms of how it is constructed, as it were, from the undefinable definers), then they are also defined by acquaintance, and so, identified by acquaintance, in my parlance.

A complex object of identification by acquaintance (such as <a,b> above) is *not* a universal, insofar as it is identified ultimately in reference to acquaintance with simple particulars, in the aforementioned way. Rather, it is a complex particular. Being a complex (being "made from" simple particulars) does not make it a universal. Russell's original distinction between knowledge by description and knowledge by acquaintance may render the knowledge of the "what is" of a complex particular as knowledge by description. (I'm not sure if it does or not.) If it does, then my use of this distinction departs from his at this point, while agreeing with his in categorizing the "what is" of a simple particular as the identification/knowledge by acquaintance. Also, if it does, then, the individuation of a particular can be surely achieved by means of such "description." For instance, we are indeed individuating a complex particular, say, S={a,b}, trivially by this "description." We can also individuate a simple particular by "description," say, a as a common element of {a} and {a,b}, because it happens to be unique. But, these identifications do not cross the epistemological gap mentioned in the Essay 1. When I claim that the identification by description cannot work across this gap, I use the notion of description quite differently from this (probably) Russellean use in this regard (though in a way recognizably continuous from it). I will clarify it shortly. In any case, let us say that simple particulars are identified (if they are identified) by *direct acquaintance*, and that complex particulars are identified by *indirect acquaintance*, if the distinction is needed. Also, we say that a complex particular is *defined* from or in terms of simple particulars, and that such is a *definition by acquaintance*.

[Note: To view the identification of an ordered n-tuple as a case of the identification by indirect acquaintance assumes that our identificational background against which we do this identification allows us to distinguish each ordered n-tuple from its permutations, in particular, its exact reversal (e.g., <c,b,a> is the exact reversal of <a,b,c>), *by acquaintance*. (And when we assume this, we also assume that we can distinguish between the identification of something as an ordered n-tuple and the identification of the "same thing" as an unordered n-tuple (i.e., a set), as a matter of difference in the level of generality.) Crudely speaking, when we hold this assumption, we are in effect assuming that we can "arrange" the same set of undefinable definers (simple particulars) differently with respect to the "orientation" of the one-dimensional Euclidean space in which they are "arranged." And, here, the said "orientation" of the space is assumed to be, in a sense, identified by us by a kind of *direct acquaintance*. This direct acquaintance, however, does not seem to be exactly the same kind as that with which we identify a and b, although the two kinds of direct acquaintance seem to share enough to be both called direct acquaintance. For this reason, the acquaintance involved in the identification of an ordered n-tuple is not exactly the same sort of acquaintance as the one involved in the identification of a set (an unordered n-tuple). The difference is important for further investigation of the nature of what I call the identification by acquaintance. For now, however, I gloss over this difference, too, to focus on the difference between the identification by acquaintance and that by description.]

3. Example of identification by description

Now, our next example. Suppose that we are instructed as follows: Let B be a set, each of $\cup$ and $\cap$ be a binary operation closed on B, C be a binary relation on B, and each of 0 and 1 be an element of B. Further, let this 6-tuple $\langle B, \cap, \cup, C, 0, 1\rangle$ satisfy the following axioms:

(1) Each binary operation is associative: for all x, y, and z in B,

$$x \cup (y \cup z) = (x \cup y) \cup z \quad \text{and} \quad x \cap (y \cap z) = (x \cap y) \cap z.$$

(2) Each binary operation is commutative: for all x and y in B,

$$x \cup y = y \cup z \quad \text{and} \quad x \cap y = y \cap z.$$

(3) Each binary operation distributes over the other: for all x, y, and z in B,

$$x \cup (y \cap z) = (x \cup y) \cap (x \cup z) \quad \text{and} \quad x \cap (y \cup z) = (x \cap y) \cup (x \cap z).$$

(4) For each x in B,

$$x \cup 0 = x \quad \text{and} \quad x \cap 1 = x.$$

(5) For each x in B, there is y in B such that

$$x\, C\, y \quad \text{and} \quad (x \cup y = 1 \quad \text{and} \quad x \cap y = 0).$$

[I'm not quite sure if it is appropriate to count each of (1) – (4) as one axiom. Each of them may count as containing two axioms. But, these do not have to concern us now. For convenience, I call (1) – (5) axioms.]

This is of course how a Boolean algebra (an abstract mathematical structure) is defined standardly. Being instructed only this much, that is, without being given a *particular* 6-tuple of mathematical objects (I'm using the term "particular" technically), we may be said to be given nothing (yet) that is what the binary operation $\cap$ or $\cup$ is, or what the element 1 or 0 is, etc., in one sense of "what is." Namely, we have to say so in the identification-by-acquaintance sense of "what is." For instance, once we are given a particular 6-tuple, $\langle P(S), \cup, \cap, C, \varnothing, S\rangle$ (where P(S) denotes the power set of the aforementioned set S, and the rest of the 6-tuple is supposed to denote the particular mathematical objects accordingly), we are given, e.g., what $\cup$ is. It is the set-operation of union defined on P(S), which is ultimately the set of triples, $\{\langle\varnothing\varnothing\varnothing\rangle, \langle\varnothing\{a\}\{a\}\rangle, ..., \langle SSS\rangle\}$. Before that, we have nothing that is what $\cup$ is, in this sense of "what is." Still, in another sense, we know a lot about what each of the 6-tuple is, just by being given the axiomatic instruction above. Our mathematical knowledge of the theory of Boolean algebras consists in this kind of knowledge. This is the sense in which I say that these operations (e.g., $\cap$ and $\cup$), elements (by 1 and 0), and everything mentioned in a theoretical discourse on Boolean algebras, are identified *by description*. As such, they are what I call universals. Similarly, until we are given a particular 6-tuple of mathematical objects, there is nothing to call a "structure" (or a Boolean algebra). And yet, we can talk about the "formal identity" (so to speak) that is shared by all the particular Boolean algebras, by virtue of which we call all of them Boolean algebras. We can arguably call *it* the *structure* of Boolean algebras, in a certain abstract sense of the term "structure." This is also a genuine universal, and is what is represented or identified, *by description*, in the above axiomatic instruction. When it comes to identification of universals, that is, identification by description, it seems much more appropriate to call such a totality (a structure in this abstract sense) simple (if anything is to be so called), and its "parts" complex, because, obviously, the latter are defined *in reference to* the former (so-defined by description, more specifically, by all the axioms taken together). In other words, it seems to be the totality that serves as the undefinable definer in the case of the identification by description. This is why I call this kind of identification top-down, while the identification by acquaintance bottom-up.

4. Identification by description on closer look

4.1 Postulates for genuine description and genuine simple universal

The above example, given in contrast to the first example (of a, b and S), hopefully gives some sense of what I mean by "identification by description" (and by "universals"). But it might have given an impression that every axiomatic mathematical theory is a genuine description of a genuine simple universal (a structure that constitutes --- not "is constituted by" --- its "parts"). Strictly speaking, this is not so. To explain this, let me first set up terminology and notation: a formulation Γ of a mathematical theory is said to define (by genuine description or non-genuine description) a mathematical structure |Γ| (a genuine simple universal or a non-genuine simple universal).

[Note: Sometimes, in mathematical discourses, the term "structure" is used for a complex mathematical particular, and the term "theory" is used for what I here call a structure, in the aforementioned abstract sense. For instance, the aforementioned *particular* 6-tuple, <P(S), ∪, ∩, C, ∅, S>, is called a "structure," and different formulations that define the same structure (in my sense) shared by all Boolean algebras are said to be different formulations of the "theory" of Boolean algebras. However, the term "theory" is also used for a set of axioms, at least in mathematical logic. This is confusing. So, in referring to Γ and |Γ|, I will use the terms a "formulation" (of a theory) and a "structure," where a "structure" is used strictly in my sense, not as a complex particular but as a simple universal --- although it may not be a *genuine* simple universal. Also, note that it is not assumed that axioms in Γ are sequence of symbols. We do not assume that axioms in Γ are written in a formal system. All that is assumed is that each axiom is a well-stated mathematical statement.]

Using these terminology and notation, I postulate as follows (to clarify what are genuinely said to be identification by description and simple universal):

Postulate I:

(Given a structure |Γ| that is defined by a formulation Γ,) Γ defines |Γ| by *genuine description* iff Γ satisfies all of the following conditions.

Condition (i): Γ contains no individual constant symbol.

Condition (ii): In Γ, each use of existential quantification is bound by a universal quantification through a relational predicate, so that the existential claim is relativized to the universal claim (e.g., "for all x there is y such that Rxy" as opposed to "there is x such that for all y Rxy"). [Note: replacing an unbound use of existential quantification by the negation of universal one (with due modification) does not change an otherwise (ii)-failing formulation Γ into a (ii)-meeting new formulation Γ'.]

Condition (iii): Γ contains no binary predicate that *has to be* interpreted as the "identity" (or "equality") in order for Γ to be a formulation of |Γ|.

Postulate II:

(Given a structure |Γ| that is defined by a formulation Γ,) |Γ| is a *genuine simple universal* iff there is a formulation Γ' (which may or may not be Γ itself) that defines |Γ| by genuine description (i.e., that defines |Γ| while satisfying all three Conditions above).

(Notice that a mathematical structure |Γ| can be defined by two or more formulations, equivalent to one another in the sense that, if Γ and Γ' are equivalent, all axioms of Γ' are theorems of Γ and all axioms of Γ are theorems of Γ'. Many mathematical theories do have more than one formulation, all being equivalent in this sense. This is why the second claim is needed.)

4.2. Boolean algebra as a genuine simple universal (and rationales for Postulates)

Now, it is obvious that the formulation of the theory of Boolean algebras given above does not satisfy the three conditions. It meets (ii), but conspicuously fails to meet (i). (For now, let me postpone the question of whether it meets (iii) or not.) However, the structure of Boolean algebras can be defined by an alternative formulation, as follows: Let B be a set, $\cap$ be a binary operation closed on B, and $'$ be a unary operation closed on B. And let the triple $\langle B, \cap, '\rangle$ satisfy the following axioms:

B1. $\cap$ is commutative.

B2. $\cap$ is associative.

B3. For all x and y in B, if there is z in B such that $x \cap y' = z \cap z'$, then $x \cap y = x$.

B4. For all x and y in B, if $x \cap y = x$, for all z in B, $x \cap y' = z \cap z'$.

From this formulation, we can prove the unique existence of two elements that meet the characterizations of 0 and 1 in the first formulation (assuming the non-emptiness of B) and prove all of its five axioms. (Without the assumption of the non-emptiness, the existence of any element of B cannot be proved, and, so, neither can the unique existence of each of 0 and 1. But this is perfectly fine for a genuine description of a genuine simple universal, as I shortly explain.)

[By the way, this second formulation is not my original, but is one apparently known for long in mathematics. I don't know who found this formulation. I personally learned about this from a textbook by Robert Stoll (called "Set theory and logic"). A proof of the five axioms of the first formulation from B1-B4 is given in this textbook. I can send a scan of this part, if anyone is interested.]

This new formulation meets the Condition (i). And, the equivalence between two formulations means that, in a sense, the individual constant symbols, "0" and "1," are not used *essentially* in the first formulation. In other words, in defining (that is, identifying) the structure of Boolean algebras, we do not have to rely on our "direct reference" by such symbols to, or our identification by direct acquaintance of, some of its elements. If such reliance is unavoidable in defining this structure, it is hard to call a genuine simple universal. This is one reason why I separate the condition for Γ to be a definition of a structure by genuine description from the condition for |Γ| to be a genuine simple universal in the way I do. My belief behind Condition (i), in this respect, is that if a structure |Γ| is a genuine simple universal, there must be a formulation Γ free of any use of individual constants.

This may be a good place to explain why I posit the Condition (ii). The main reason is that unbound existential claim makes a real reference to a particular. It is an indefinite reference (a reference to an indefinite particular), as opposed to the definite reference made by an essential use of an individual constant symbol. But, it's a real reference, nonetheless. Universal quantification and existential quantification bound by a universal quantification through a relational predicate do not make such a real reference. What they make is a kind of pseudo reference or hypothetical reference, by means of which to characterize "primitive" relational predicates. That is, they merely relate a number of such predicates one another through the use of variables that are rendered co-referential by (and under) the binding quantification, so that these predicates co-define one another to a certain extent. Of course, these relatings themselves --- that is, the sentences (i.e., the axioms) each of which relates a number of relational predicates by quantificationally co-referential variables --- get related one another through repeated use of these "primitive" predicates across them. This way, a set of axioms completes the definition of a structure *at the same time*, through the bonding of all the partial mutual definitions into one whole. How an axiomatic definition of a structure works is essentially the same whether the definition meets the Condition (ii) or not (as long as it meets Condition (i) and (iii) --- a definition that fails to meet either of them cannot be said to define *everything at once* which I take to be a minimal condition for a definition by description). But, if it meets (ii), once each axiom draws up one partial mutual characterization of a number of "primitive" predicates, the co-referentiality maintained by the topmost quantification finishes its role, and, since it is universal quantification, with its finishing its role, the pretense of reference is altogether *over*--- so to speak. With the topmost quantification being universal, this kind of hypothetical reference is really a mere ladder to throw away once its job is over. If there is a topmost quantification in an axiom that is an existential quantification (thus it fails (ii)), however, the co-reference-maintenance is gone at the completion of this quantification, but a fact of reference *to a particular*, remains --- so to speak. Such a description cannot be genuine description in my view, and, if this reference to a particular cannot be eliminated from a description of a structure, the structure cannot seem to be a genuine simple universal. This clarifies what ultimately motivates the stipulation of Condition (i), which the stipulation of (ii) shares: A genuine description of a genuine simple universal should not or cannot rely on a real reference to a particular.

Digression: Some consequences of Postulates

This may also be a good place to explain one consequence of the Conditions (i) and (ii): If a formulation Γ meets (i) and (ii), then it contains no monadic (one-place) predicate, except for the completely trivial use of it, in the form of: "For all x Fx." The reason goes as follows. First, Γ contains no individual constant symbol, by (i). So, if it contains a monadic predicate F, each occurrence of it in Γ predicates of a variable x, in the form of Fx. Since each axiom in Γ is a well-stated mathematical statement, it is closed, and, so, each such Fx is bound. But, what bounds it cannot be existential quantification, by (ii). So, each such Fx occurs ultimately in the form of "For all x Fx." Given that F is a "primitive" notion in Γ, any such use of F is completely trivial. This completes the derivation of this consequence from (i) and (ii).

Now, this implies something important. According to this epistemological conception of the universal/particular distinction, *there is no genuine universal that is monadic. All genuine universals are relations, or, a.k.a. relatives.* Today, when metaphysicians dispute over whether universals are real or non-real (or nominal or non-nominal), their focus seems to be placed on monadic universals. But, it seems to occur seldom to them that the very notion of a *monadic universal* may be an epistemologically illicit notion, a sort of oxymoron. Here, my claim is that it is. But, this is probably a very controversial claim. If I can succeed to make some metaphysician friends of mine take this possibility seriously, I would count it as a success enough for this series of essays. (Perhaps, I have to make a room for an exception. We may call a genuine simple universal a monadic universal, if we allow us to use the term "monadic" somewhat liberally. But this is of course not the kind of a universal that metaphysicians today call a universal.)

[I think I am in a sense correct in pointing out the oxymoron-ness of the notion of a monadic universal, although I think I am making a self-contradiction between what I say and what I do, in pointing it out. After all, with me using the language here in the way I do, I in practice commit myself to a view that the claim I'm making is true, not false. Here, "true" and "false" are treated as monadic universals of a meta-linguistic sort. My deed contradicts my word. (And, this is another reason why I am suspicious of the "coherence," or a kind of epistemological soundness, of our natural conception of truth.)]

There is one more consequence of the Conditions (i) and (ii) that is worth a mention. A genuine simple universal is such a structure which takes the empty set --- conceived of as equipped with all relevant relations, though each of which is empty --- as a null model (a null complex particular that shares the structure). Why this follows from (i) and (ii) should be obvious. But I also think that this is just how a genuine simple universal should be. Since a genuine description of such a structure should be free from any real appeal to a particular, any such structure should countenance an empty set as its null model.

[Note: Above, I explained the two consequences of (i) and (ii) as consequences, thereby probably making an impression that the stipulation of (i) and (ii) was motivated independently from them. This is a mere rationalization I adopted for the sake of exposition. The ideas that a genuine simple universal, a structure that is to be described by a genuine description, ought to be described solely by relations of relations and ought to have the empty model, are as much a motivation for (i) and (ii) as the motivation mentioned above. That is, I came to settle with the formulation of the Conditions (i), (ii), and (iii) in part because these consequences follow from them. I believe that this is one legitimate way of doing philosophy. Sometimes many philosophical ideas present themselves simultaneously as inter-related pieces of puzzles to be put together into a whole. Which ideas to present as principles and which as consequences is merely a matter of exposition, and of no real importance, in such a case.]

Back to 4.2.

Now, someone may think that both formulations of the theory of Boolean algebras fail to meet the Condition (iii) --- especially if he or she thinks that without identity, majority of mathematics is rendered impossible. Whether this notion is true or not depends on how to understand "majority," of course. But this seems hard to accept. Nowadays, mathematics seems mainly the kind of abstract mathematics in which a theory (or more precisely, a structure in my sense, something identified by a formulation of a theory) is distinguished from its models (particular mathematical complex objects (e.g., $\langle P(S), \cup, \cap, C, \emptyset, S\rangle$)). The notion of "identity" *as such* belongs to the realm of models, that is, particulars. In principle, or in the spirit of this theory-model distinction, no theory (i.e., formulation of a theory) Γ should involve such a binary predicate symbol that *has to be* the identity for it to be a definition of $|\Gamma|$. In any case, if a formulation Γ in fact contains such a symbol, there cannot be two models M and M' of $|\Gamma|$ such that there is a non-isomorphic homomorphism from M to M'. But, many mathematical theories do admit of non-isomorphic homomorphism, as far as I know. Probably this is a hallmark of this kind of abstract mathematics. [By the way, strictly speaking, both a model and a homomorphism can only be defined relative to a specific formulation Γ, i.e., axiomatic definitions of relations/operations (or symbols thereof). So, saying that a structure has a model or admits of homomorphism is not exact. Allow me to abbreviate the manner of expression here.] So, my (admittedly lay) impression is that the part of mathematics that really is rendered impossible by losing the identity won't count as "majority" of mathematics, although that part is surely very important part of mathematics. [Honestly speaking, I earlier wrote, in earlier drafts of this Essay, that: *"majority" of mathematic remains perfectly possible without the particularistic notion of identity (except for its heuristic use, just as recourse to models is freely made in such mathematics for the same heuristic purpose).* But I was wrong. Both kinds of mathematics (one that can survive without identity, the other that cannot) are crucial for mathematics as a whole. They are crucial because the exact relation between them is probably a crucial part of the answer to the question, "What is mathematics?", to be pursued by philosophers.] All that mathematics of this abstract kind needs is the notion of a *congruence relation*, defined relative to a given formulation Γ on a model M --- i.e., an equivalence relation **=** on M such that for any x and y in M, if x**=**y, then x and y are indiscernible by the "primitive" vocabulary of Γ. I don't think that excluding this part of mathematics allows the claim of "majority" to the rest of mathematics. But anyway, the theory of Boolean algebras does admit of non-isomorphic homomorphism, and, so, both formulations of the theory above meet the Condition (iii).

[I must qualify my claim. The aforementioned abstract kind of mathematics does indirectly rely on the particularistic notion of identity when it concerns itself with the consistency of a formulation of a theory, i.e., a set of axioms. A set Γ of axioms is standardly tested for its consistency by finding a model, a complex mathematical *particular*. Our conception of such a complex particular seems to involve an unavoidable reliance on the particularistic notion of identity, or an unavoidable reliance on our ability to identify *particular* elements of a structure by direct acquaintance. (I believe that what are relied on in these two cases are "equivalent" in some sense.) This is one reason why I think that the notion of consistency, along with that of truth, has to be put to a careful philosophical investigation with an eye toward a re-thinking of the descriptive/normative distinction, to which it has never been put, at least to the best of my knowledge.]

Having introduced the notion of congruence relation, I now can, and should, revise the Condition (iii) a little more precisely. Originally, (iii) says:

Condition (iii): Γ contains no binary predicate that *has to be* interpreted as the "identity" (or "equality") in order for Γ to be a formulation of |Γ|.

Here, the phrase "for Γ to contain a predicate" sounds to mean "for Γ to contain a predicate symbol." (Indeed, in the Condition (i), the analogous phrase is supposed to be taken that way.) But, if this phrase taken in that way, there is a vacuous way for a certain Γ to meet (iii); namely, a Γ that contains no binary predicate to begin with meets (iii) vacuously. The Condition (iii) is not really meant allow that. To fix this, let us first note that for any formulation Γ of a structure |Γ|, there is one and only one congruence relation γ. [To be precise, I should state this more carefully. But a precise explanation seems tedious. Assuming that most people who can read this Essay probably can understand what I mean, let me give only a coarse outline. If "two" semantically distinct γ' and γ'' (defined on two distinct models M and M' of Γ, respectively) are two extensions of one syntactic definition γ, they count as identical, in this context. If "two" syntactically distinct γ' and γ'' (defined by two distinct syntactic definitions) share the same extension γ in every model M of Γ, they count as identical, too.] So, if we are given a formulation Γ of a structure |Γ|, we are also given the unique Γ-congruence relation γ.

Using this notion of γ defined relative to Γ, (iii) is hereby revised as follows:

Condition (iii'): The Γ-congruence relation γ does not have to be interpreted as the "identity" (or "equality") in order for Γ to be a formulation of |Γ|.

Notice that the above explanation of how two formulations of the theory of Boolean algebras meet the Condition (iii) is not affected by this revision. And now, with this revised Condition (iii'), I can say that this explanation should have also explained why I posit this Condition: The notion of identity as such only belongs to the realm of the particulars. If a structure |Γ| is a genuine simple universal, no formulation Γ of |Γ| can have γ that has to be interpreted as the identity.

Notice that the Condition (iii') is distinct in this regard from (i) and probably (ii). Finding, of one formulation Γ, whether it meets (iii') or not is enough to conclude that the defined structure |Γ| is a genuine simple universal or not. This is not the case with the Conditions (i) or (ii). (I'm not as confident on this about (ii) as I am about (i), though.) (iii') is this way because of the following: A formulation Γ meets (iii') iff it admits of non-isomorphic homomorphism. But, if one Γ admits of non-isomorphic homomorphism, all equivalent Γs do, trivially. (That is why I can use the sloppy language of a "*structure* |Γ| admitting/not admitting of homomorphism.") The converse is even more trivially true. So, one Γ admits non-isomorphic homomorphism iff all equivalent Γs do. So, one Γ meets the Condition (iii') iff all equivalent Γs do. As far as I'm concerned, it is because (i) and (ii) that I had to separate the condition for a formulation Γ to be a genuine description from the condition for a structure |Γ| to be a genuine simple universal, in the way I did.

Digression again: Another consequence

[Note: There seems to be a certain relation between Conditions (i) and (iii'). This is only a conjecture at this point, but seems worth a mention, although it is not necessary to understand what I mean by a genuine description of a genuine universal. I'm inclined to conjecture as follows: **Suppose that there is a formulation Γ of a certain structure |Γ| which contains individual constant symbol(s), "e_1," "e_2," ..., "e_n." There is an alternative formulation Γ' of |Γ| in which no individual constant is used iff there is a homomorphism h: M → M' (where M and M' are distinct models of Γ) such that for each element ei' in M', the set {x∈M|hx=ei'} contains more than one element.** I have no proof. But, the right-to-left conditional seems to have to be true. The left-to-right does not seem so, but I cannot think of a counterexample. This may be only because of my ignorance in mathematics. I'll appreciate if someone tells me otherwise.]

Back to 4.2.

By now, I hope that it is clear that the second formulation of the theory of Boolean algebras above meets all three Conditions (with (iii) being clarified as (iii')). So, it is a genuine description of a genuine simple universal, which is the structure of Boolean algebras.

4.3. Most mathematical structures (including group) as not genuine simple universals

So far, Boolean algebra is the only structure I can confidently claim to be a genuine simple universal. There may be more structures of this sort. But, my mathematical background is so little and my mathematical talent is so limited, that I cannot confidently claim about any other structure that it is a genuine simple universal. In fact, I suspect that the structure of Boolean algebras may be the only structure of this sort.

Take the structure of group theory, for instance (for this structure, being one of the most abstract structures in mathematics, seems to be an obvious candidate to suspect as a genuine simple universal). Its standard formulation goes as follows: Let G be a set, and • be a binary operation defined on G. Let the pair <G, •> satisfy the following axioms:

G1. Closure: For all x and y in G, x • y is also in G.

G2. Associativity: For all x, y and z in G, (x • y) • z = x • (y • z).

G3. Identity: There exists e in G such that for every x in G, e • a = a • e = a.

G4. Inverse: For each x in G, there exists y in G such that x • y = y • x = e.

Here, we disregard G1 as miscounted as an axiom due to the sort of conceptual "liberty" or "looseness" enjoyed by mathematics, which is a vice from a certain meta-mathematical point of view but is harmless and even vital for the practice of mathematics. (Just in case: By "practice of mathematics," I do *not* mean applied mathematics. The practice of pure mathematics is meant.) That is, the notion of whether a given operation is closed on G makes sense only on the presupposition of some background set, say, U, of which G is considered a proper subset. Moreover, as such, both U and G are essentially conceived of as complex particulars. In defining a structure *by description*, the notion that each relation is defined on the assumed totality (hence, if some relation is given under the guise of operation, the notion that it is "closed on it") is *presupposed*, and, so, is not even statable. So, counting G1 as an axiom on a par with G2-G4 is a case of the familiar conceptual "liberty" granted in the practice of mathematics, as found in their concern with the consistency of a "theory" (conceived "loosely" as regards whether it is a set of sentences or something that is defined by it, and if something defined, by which kind of definition, by description or by acquaintance). [Of course, to say this is by no means a criticism of the practice of pure mathematics as practiced today, *qua* practice of pure mathematics. What I mean by the "granted liberty" includes the notion that shifting back and forth between the realm of universals and the realm of particulars freely (but in accordance with some unspecifiable "rules" --- or so it seems to "us" inevitably) is the vital core of pure mathematics, as a semiotic practice of "us." Still, this does not prevent me as a philosopher to point out that G1 is not on a par with G2-G4 in this regard.]

Focusing on G2-G4 now, this definition meets the Condition (i). And, the defined structure is known to admit non-isomorphic homomorphism. So it meets (iii'). (Some might think that this formulation involves a use of "e" essentially as an individual constant symbol, at G4. But, it looks that way only because this formulation allows us to prove the "uniqueness" of the identity element, e, as defined in G1-G3, where this proven "uniqueness" is *only relative to* the congruence relation =.) Furthermore, it contains no monadic predicate. However, this formulation fails to meet (ii). G3 contains an unbound existential quantification, and appears to involve a real reference to an indefinite particular. And I don't know if this unbound existential quantification can be eliminated from a formulation of the structure of group theory. I doubt that it can be.

For this reason, I suspect that majority of known mathematical structures are not quite genuine simple universals. But, even if so, some structures seem to be more nearly a genuine simple universal than others. So, for instance, the structure of group theory seems to fail to be a genuine simple universal only because its formulations cannot meet the Condition (ii). By contrast, the structure defined by the standard formulation of Peano arithmetic seems more of a disguised complex particular than a simple universal. I believe that this structure cannot be defined without breaking the Conditions (i) --- hence, in spirit, (ii) too --- or (iii').

But anyway, this concludes this Essay 2. I hope that what I mean by identification by acquaintance and identification by description, as well as by particulars and universals *as objects of identification*, are clear now. I will appreciate questions and objections.

5. Postscripts

I decided to change my terminology. Instead of talking about epistemological "illicitness" etc., I'll talk about epistemological *amphibiousness*.

I have used various derogatory terms to characterize our purported "acts" (illocutionary acts) that purport to work across the fundamental epistemological gap: illicit, incoherent, wrong, improper, etc. But, at the same time, I repeated (probably annoyingly often) that "our" own existence (as thinking/talking beings) depends on our socially committing to these "errors" in our "acts," or that these "errors" are pragmatically inevitable for "us" to commit in "acting," or that any attempt by "us" to deny the "possibility" or "reality" of such "epistemologically erroneous" "acts" (which is an "act" if it is so much an attempt to do something at all) makes "us" commit a self-contradiction between what it says and what it does. Although these "errors" are indeed "errors" from a certain philosophical point of view (which I try to express by the term "epistemology" when I call those "errors" "epistemological" errors or when I call the gap between the realms of universals and particulars the fundamental "epistemological" gap), calling them "errors" just does not sound right, or does not make me comfortable. So, I adopt the term, "epistemological amphibiousness."

So, for instance, I would say that one purpose of my re-thinking of "mind" is to prove (in some sense) that the notions like "truth" and "consistency" are epistemologically amphibious.

By contrast, the epistemologically "ideal" sort of "act" that does not commit the amphibiousness will be called epistemologically pure. According to my re-thinking, there are two kinds in such "act." Whether "we" can say or think that "we" are capable of either of them is a question I'm still investigating.