Why not naturalize the epistemology of mathematics?

Seiichiro Yasuda

Apology

The present paper is a message to those who are interested in the contemporary trend of naturalized epistemology, either critically or sympathetically. Yet, it makes no reference to the literature. The reason for this goes back to the fact that the essential idea behind the message has been developed quite independently of naturalized epistemology. More specifically, the idea has been developed, over the last two decades or so, through my reflection on what language, logic, and mathematics are (among other things), which has always meant, for me, about what we are (at least in one respect). More than half of this period, I have been ignorant about the post-positivistic trend, and most of the rest (ever since I learned about it up to almost now), I have had no sympathetic interest in it. So, I knew virtually nothing about the literature when I recently realized that my idea (about language, logic, mathematics, etc., and about ourselves) might be seen, and/or presented, as a call for a new (?) version of naturalization of epistemology, and that a fragmentary hint of this "new" kind of naturalized epistemology might be found in Quine's classical 1969 paper, "Epistemology naturalized" (hereafter EN), or possibly in Neurath's even earlier philosophy of science as well, expressed by a simile known as Neurath's boat. In fact, to explain this realization is the purpose of the present paper. Ideally, I should have done literature study before I try to publish about this "new" naturalized epistemology, to see if it's really new in the literature, which I have not done. My neglect of this intellectual duty is for a sole reason that I might run out of time if I wait writing about it until I complete literature study sufficient for publication. I apologize for this selfish and amateurish neglect.
1 Introduction

When Quine's "Epistemology naturalized" (1969, hereafter EN) is recalled today as a major point of departure for the contemporary trend of naturalized epistemology, it seems to be usually recalled for its advocacy of empirical-scientific methodology for epistemology in general, in condemnation of the purely rational approach of traditional epistemology. However, a good part of the condemnation in EN was actually geared toward a peculiar brand of the traditional approach, namely, the logical positivist's rational-reconstructive approach, as it is applied to a particular branch of epistemology, namely, the epistemology of the natural science of physics. On this ground, it seems possible to interpret his call of the naturalization of epistemology as containing an aspect of a specific criticism of this peculiar brand of traditional epistemology as it is applied to this particular branch of epistemology --- excluding, at the very least, the epistemology of mathematics from its intended scope. Assuming that it does contain such a specific criticism, this relatively neglected aspect of EN is precisely where I would find its greatest relevance, both to contemporary epistemology and cognitive psychology (the latter being broadly construed as naturalized epistemology) and to the history of these disciplines (including the issue of Quine's role or place in this history).

I suspect that my claim of contemporary relevance of this anti-reconstructionist aspect of EN might puzzle at least some people who are interested in the contemporary naturalized epistemology, either critically or sympathetically. That is to say, I expect that it would puzzle those who think that EN's contemporary relevance, for better or for worse, lies primarily in its general methodological criticism of the purely rational traditional epistemology. From this point of view, the anti-reconstructionist aspect of EN would surely look pointless, because the rational-reconstructive approach is only a very special brand of the traditional rational methodology, rejecting which by no means implies or motivates the adoption of empirical-scientific methodology in general
epistemology. To give a brief outline of my response to this puzzlement: I think that while such a general methodological criticism can be found in EN, it represents a backward, rather than progressive, side of Quine's naturalism, that is, a lingering and blinding effect of a fundamental assumption of the traditional epistemology --- from which even Quine the naturalist could not completely free himself. If some people are puzzled by my claim above, that is most likely because they too are being blinded by this traditional assumption. (My response ends here.)

In the present paper, I try to substantiate this response by explaining the claimed contemporary relevance of the anti-reconstructionist aspect of EN. This explanation will consist of delivering roughly two theses: (i) that the logicist's proud achievement in the epistemology of mathematics, namely, the set-theoretic foundation of mathematics, is also a rational reconstruction, and, thus, should have been a part of what Quine the naturalist tried to leave behind as history;\(^1\) and (ii) that we should re-conceptualize the notion of the naturalization of epistemology by reflectively transcending the fundamental assumption of the traditional epistemology, which may be described as an unreflective distinction between rational and empirical methods of inquiry (by which I also mean, inclusively, the distinction between rational and empirical truths);\(^2\) which transcendence, and

\(^1\) Or rather, he should have ceased to take it at face value (as a positive result for the genuine epistemology of mathematics), and should have taken it as an informative episode of the history of our kind of beings. --- What I mean by this will be gradually clear in the present paper.

\(^2\) As an amateur historian of the intellectual trend of which my own philosophy is a product, I have been pursuing a hypothetical view of the post-positivist phase of Anglophone philosophy broadly as a phase of growing trend toward this reflectively transcendence of this unreflective epistemological distinction. (Actually, my view is multifaceted, and this is just one of several possible ways to describe it.) Moreover, observing the principle of charity (in my own way), I'm also working on an interpretation of Quine's overall philosophical outlook (including his epistemological naturalism) as a transitional scene in this historical trend. From this historical perspective, Quine's general methodological criticism --- of the purely "rational" method of the traditional epistemology, in contrast to the
ultimately only that, can render intelligible my thesis (i). I will explain these two theses basically in this order.

Before embarking on this explanation, an apology is in order, however. According to my own claim, understanding the thesis (i) presupposes understanding the thesis (ii) as its necessary prerequisite. Indeed, I think that those who are puzzled by my claim of the contemporary relevance of Quine's anti-reconstructionist criticism would be even more puzzled by (i), until (ii) is explained. However, in the atmosphere of the contemporary epistemology, the content of the thesis (ii) seems to be somehow extremely hard to come across. And, I have a hunch that one of the greatest hindrances here is the mesmerizing "success" of the logicist's set-theoretic foundation of mathematics, which has been almost ubiquitously taken as a positive result in the epistemology of mathematics, rather than as a mere rational reconstruction. That is to say, I have a hunch that, at least for some segment of today's audience, facing the thesis (i) before facing (ii) could have an effect of removing or reducing certain "psychological" block in their way to proper understanding of (ii). So, in the present paper, I choose to explain the thesis (i) before (ii). But, as I stated above, the thesis (i) presupposes the thesis (ii), in content. So, although I will do my best so as not to presuppose anything that is yet to be explained, my effort will inevitably fall short of the ideal. I must apologize for this compositional defect in advance. Moreover, the two theses, even taken together, make up only an integral part of a systematic whole of my current view of "how things in the broadest possible sense of the term hang together in the broadest possible sense of the term" (Sellars, 1962), in which each constituent thesis presupposes all the others. (I may liken this situation to an axiomatic definition of an abstract mathematical concept, such as group.)³ In what

³ "empirical" method of natural science --- is surely a backward side of his naturalism, for it presupposes this very distinction.
³ By adding the adjective "abstract" here, I mean to exclude concrete mathematical concepts such as set, number, point/line/plane, etc., which we have concrete mathematical intuitions about, and which, if "axiomatized,"
follows, I will also do my best to make the present paper readable independently of the other related theses of mine. But, my effort will also fall short of the ideal in this regard. I apologize for this in advance, too.

One more note may be in order, so as not to mislead my readers to think that the present paper only delivers to technical and pedantic interests concerning the history and methodology of academic epistemology, as a branch of academic philosophy. If I'm right, the alleged contemporary relevance of the anti-reconstructive criticism of Quine, to be explained in this paper, is universally relevant to everyone of us --- i.e., every linguistic being. I mean, the ultimate argument of the present paper, which may be summarized as the thesis that we should cultivate a philosophical perspective from which to reflectively transcend our own unreflective distinction between rational and empirical methods of inquiry, is such that it is destined, due to indexical sort of necessity, to be relevant to you, whoever you are, regardless of any indexically non-necessary and thus irrelevant factors such as whether you are a terrestrial or extra-terrestrial being and whether you are a biological or non-biological being. All that matters is the

end up rendering the resulting sets of "axioms" (or schemata thereof) as axiom-sets in the foundational sense of the term, as opposed to the structural sense. (See Feferman 1999, for a little more explanation of this distinction.) My analogy of an axiomatic definition is meant to invoke only the image of a structural axiom-set which defines an abstract mathematical concept, not that of a foundational axiom-set which does not define anything in that sense, but, rather, lays a rational foundation for a system of concrete mathematical concepts, as we use them and we "know" of them in some sense. In passing, it may be worthwhile, as a preparation for what to come later, to note that the former "definition" is a linguistic act which works in the so-called world-to-word direction of fit (where the "world" refers to the world in which mathematics takes place, "as an institution or process in the world," to borrow a phrase from EN, p. 84), while the latter "laying of a rational reconstruction" is a contrastive sort of linguistic act which works in the word-to-world direction-of-fit (where the "world" refers to the "mathematical world," about which mathematics is purported to be.)  

4 Actually, these distinctions would be also denied as naïve and non-reflective distinctions by the reflective/transcendental kind of naturalism of the present paper, insofar as they are considered in relation to
pragmatically inevitable fact that you are reading this now --- or more generally, receiving the message of this text through some linguistic medium (via mediation of the language-ideologically loaded and loading meta-linguistic practice called "translation," which should have happened sometime between my writing of this and your "reading" of this, as an indispensable link in the causal-historical chain of events between the two) --- and that, therefore, you are one of us in this obvious indexical, that is, pragmatics-centrically linguistic sense of the term. Moreover, I think that the ultimate argument of this paper is also destined to be centrally relevant

the question of what we are. My appeal to them here is only for its expository merit of making clear the pragmatics-centric style of philosophization, of which my claim of the universal relevance (of the ultimate argument of the present paper) is an integral part.

5 Note that, by claiming that the present paper is universally relevant in this sense, I in effect attribute a related pragmatic sort of universal relevance to the set-theoretic foundation of mathematics, despite my claim that it is a mere rational reconstruction. This attribution is intended. I think that the developments by some of us, in the indexical sense of the term, of (i) mathematics as we know it today, and (ii) this sort of set-theoretic reconstruction of our mathematical knowledge (as to concrete mathematical concepts --- see the footnote 3) are both the fate of our kind of beings, due to certain pragmatic precondition of our kind of beings. (I sense that there is some fundamental difference between the sense in which (i) is said to be the fate of us and the sense in which (ii) is said so. But, I cannot specify the difference right now. Clarifying this difference is one of future tasks of my philosophical project.) So, if Frege-Russell-Whitehead had not done this reconstruction, I think someone else would have done it, at around the same phase of the natural history of our kind of beings. Essentially the same applies, I think, to Carnap's attempt to do in the epistemology of physics what the logicians collectively achieved in the epistemology of mathematics, as well as to Quine's naive-naturalistic criticism of Carnap's project. If they had not produced their respective historical texts, someone else would have done the essentially same. Finally, I think the same applies to the argument of the present paper, too. (So, I expect that someone will criticize my philosophy as "transitional" from a point of view which is beyond my current imagination, just as I criticize all of the historical figures/texts mentioned above as "transitional" from a point of view which I think is beyond the imagination of their times.) If I'm right, the reflective/transcendental re-conception of naturalized epistemology can offer a broad explanation for all of these claims of mine.
to any being "who" can be interested in the perennial philosophical question of what we are, and to any being "who" can be interested in another perennial question of what knowledge is (where the concept in question is understood in the sense of knowing-that), and, finally, to any being "who" can be interested in yet another perennial question, hovering between the first two, namely, that of what truth is (where the concept in question is understood as a condition of knowledge in the sense of knowing-that) --- where the interests are all of the attitude of "our problem," from the point of view of the shared first-person plural perspective. In this way, the argument of this paper is meant to be a contribution not just to the history and methodology of academic epistemology or cognitive psychology, but also, more straightly, to the contemporary epistemology/cognitive psychology of physics, of language, of mathematics, and of logic (among other things). Indeed, our distinction between the historical and non-historical (or analytic) inquiries, too, will be called into question by this argument.

I start with an explanation of the thesis (i). In particular, I begin by offering a general description of the rational-reconstructive approach to the epistemology of a systematic body of knowledge of ours, to clarify in what sense such an approach should be condemned, or, for what purpose such an approach should be abandoned as irrelevant. Then, I will argue that the set-theoretic foundation of mathematics is a rational reconstruction of our mathematical knowledge, based on that general description. If my discussions up to this point are reasonable, readers will be convinced that the currently standard epistemology of mathematics, accepting the set-theoretic foundation at face value, should be naturalized in a broadly Quine's sense of the term. Hopefully, this will motivate them to know more about precisely what I mean by this "naturalization," for the common conception of this notion just won't do when it comes to the epistemology of our mathematical knowledge.
References

